Experiment 5: Interference of Light

Purpose: To observe interference effects in light reflected by a crude diffraction grating and to determine from the resulting interference pattern the wavelength of the laser source. This experiment involves multiple-source interference and least-squares analysis.

Reference: A. L. Schawlow, Amer. J. Phys. *33*, 922 (1975) [attached].

Apparatus: HeNe laser of several mW, steel ruler, meter sticks, and a least-squares-fitting program entitled LAMBDA.

Background: A reflection grating is composed of a large, uniform array of reflecting surfaces. Figure 1 depicts a flat reflection grating viewed sideon; in the present context one can think of the device as a flat array of highly reflecting plateaus separated by thin grooves. When illuminated by laser light, these plateaus behave like a collection of identical, coherent, monochromatic sources. The resulting superposition of light on the screen shows a dramatic sequence of interference maxima (bright spots) separated by darkness (where destructive interference prevails). The purpose of this experiment is to infer from the locations of the maxima the wavelength of the incident light. Since you will be using a finely-ruled steel ruler as a grating, you will be measuring the wavelength of light with (of all things) a ruler!

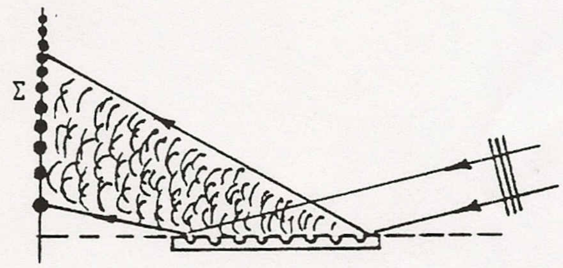

Fig. 1. Incident and reflected waves from a simple reflection grating.

Figure 2 suggests the experimental arrangement. A beam from a HeNe laser strikes a horizontal steel ruler at a grazing angle α. Interference maxima are observed at heights y_n ($n = 0,1,2,...$) on a vertical screen Σ. Let each maximum be located at an angle β_n such that $\tan \beta_n = y_n/x_o$, where x_o is the distance from the illuminated portion of the ruler to the screen. Arguing on the basis of light rays reflected from adjacent plateaus, we conclude that interference maxima will be located at heights y_n for which the difference in optical path lengths LAy_n and LBy_n between the laser L and the points y_n is an integer number of wavelengths $n\lambda$. Figure 2 suggests that this path difference is $d(\cos \alpha - \cos \beta_n)$. Hence the criterion for an interference maximum "of order n" is

$$n\lambda = d(\cos \alpha - \cos \beta_n) \ . \quad (1)$$

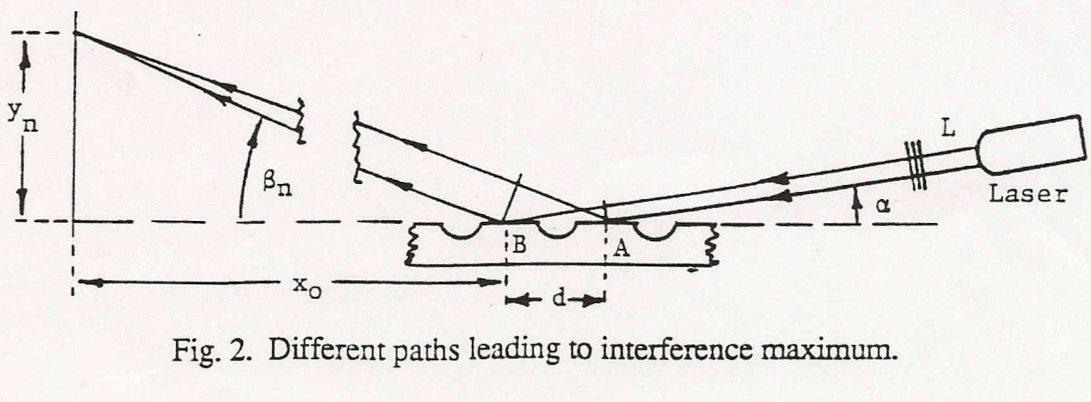

Fig. 2. Different paths leading to interference maximum.

In the attached article, Nobel laureate A. L. Schawlow shows that Eq. (1) can be written as

$$n\lambda = d(y_n^2 - y_o^2)/2x_o^2 = dz_n/2 \qquad (2)$$

where $z_n = (y_n^2 - y_o^2)/x_o^2$. Equation (2) is the working equation for this experiment.

Procedure: Place the ruler 10 or 20 meters from the wall, and position the HeNe laser 5 or 10 meters beyond the ruler. Allow part of the laser beam to pass undeflected alongside the ruler to establish a "reference spot" on the wall. Mark the location of this spot. Next you must find the "zeroeth-order" maximum (corresponding to n = 0 in Eq. 2) located at y_0. This particular maximum is the red spot where a *purely reflected* beam hits the wall. Since the identification of this maximum can be confusing, it helps to momentarily slide the ruler sideways so that the laser beam reflects off a perfectly smooth portion of the ruler. Under these conditions one gets only the zeroeth-order maximum; carefully mark its location on the wall, and then move the ruler back to its proper position. Now, by the laws of geometrical optics, the *midpoint* between the "reference spot" and this zeroeth-order maximum must lie in the plane of the ruler. All measurements of the y_n should be made from this midpoint. We recommend that you start with y_0 and not worry about the few "negative-order" maxima below it.

Adjust the laser beam and ruler in order to optimize the sharpness of the maxima. Then carefully measure the heights y_0, y_1, y_2, ... for as many maxima as you can distinguish. Think carefully about the criteria for identifying the "most appropriate point " within each maximum to call y_n and whether these criteria can be applied uniformly to all the maxima.

Once you have acquired measurements of various y_n, x_o, and d, submit these data to the least-squares-fitting program LAMBDA. This program computes the various z_n and then uses least-squares analysis to fit the optimum straight line to the z_n versus n data. From the slope of this straight line, the program determines the wavelength λ of the laser source. Compare LAMBDA's value based upon your data to the accepted value of 632.817 nm.

Measuring the Wavelength of Light with a Ruler

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(Received 29 June 1965)

A simple lecture demonstration is described, in which light from a gas-discharge laser is diffracted at grazing incidence by the rulings of a steel scale. The wavelength of light is obtained by measuring the pattern spacings and the distance from the ruler to the screen.

IT is well known that a reflecting grating with widely spaced grooves gives good diffraction spectra if the light is incident nearly parallel to the surface of the grating. Thus, several orders of reflection spectra may be seen by sighting along a good ruled scale at a small, distant light. It is even possible to estimate the spacing of the diffraction orders by viewing them against a background with scale markings, and so to measure the wavelength.[1]

However, with a small helium–neon gas-discharge laser, a good Fraunhofer diffraction pattern from a ruler can be projected for viewing by a large class. The arrangement is shown in Fig. 1. A good machinists' steel scale is positioned approximately horizontally in front of the laser, so that the light beam passes nearly parallel to, and just above one of the finer sets of rulings. The laser is then tilted downward so that part of the beam grazes the last few inches of the rulings. Several sharp diffraction orders are then seen on the screen. If some of the beam passes over the end or side of the ruler, the direct beam can also be seen. The spots from the direct beam and the zero-order diffracted (specularly reflected) beam define the angle of incidence, and so no direct measurement of angles is needed. All information required to measure the light wavelength comes from the spacings of the spots on the screen, and the distance from screen to ruler. These can be measured with another ruler, or even with the same one if the spot positions are marked.

Figure 2 shows the angles and distances involved. If i and θ are on opposite sides of the normal, and are both taken as positive quantities, the grating equation may be written

$$n\lambda = d(\sin i - \sin\theta),$$

where n is an integer, λ is the wavelength, and d is the spacing between rulings. It is more convenient to use the complements of these angles, i.e., $\alpha = 90° - i$, $\beta = 90° - \theta$ so that the equation becomes

$$n\lambda = d(\cos\alpha - \cos\beta).$$

In the experiment, the distance to the screen x_0, and the distance between spots on the screen are measured. Since $\alpha = \beta_0$ (the zeroth order is specularly reflected), the intersection of the plane of the grating with the screen lies half way between the spots of the direct beam and

[1] This was pointed out to me some years ago by Professor R. R. Richmond, University of Toronto, who remarked to some students on an appropriate occasion "If you don't behave, I'll make you measure the wavelength of light with a ruler."

the zeroth order diffracted beam. Take this point as the origin for measuring distances along the screen. Then the intersection of the direct beam is at $-y_0$, of the zeroth-, first-, second-, $\cdots$-order diffracted beams are at y_0, y_1, y_2 $\cdots$. For any of these, $\tan\beta = y/x_0$; but β is small, so that $\tan\beta \approx \sin\beta$,

$$\cos\beta = [1-\sin^2\beta]^{\frac{1}{2}} \approx [1-(y/x_0)^2]^{\frac{1}{2}}$$

$$= 1 - \tfrac{1}{2}(y^2/x_0^2),$$

$$\cos\alpha = \cos\beta_0 = 1 - \tfrac{1}{2}(y_0^2/x_0^2),$$

$$\cos\beta_n = 1 - \tfrac{1}{2}(y_n^2/x_0^2),$$

$$n\lambda = d(\cos\alpha - \cos\beta_n)$$

$$= d\left(1 - \frac{1}{2}\frac{y_0^2}{x_0^2} - 1 + \frac{1}{2}\frac{y_n^2}{x_0^2}\right) = \tfrac{1}{2}d\left(\frac{y_n^2 - y_0^2}{x_0^2}\right).$$

Thus, $(y_n^2 - y_0^2)$ is linearly proportional to n.

Since the spot spacings increase as d decreases, a widely spaced pattern is obtained by diffraction from a fine scale, such as $\frac{1}{64}''$ or even $\frac{1}{100}''$. It is worth demonstrating that coarser scales give finer patterns. One good way to show this is to slide the ruler a short distance sideways. Then part of the beam is diffracted from the coarser markings used to set-off every second, fourth, or fifth division of the fine scale. Extra spots appear between those seen originally; they are higher orders from the widely spaced long marks on the scale.

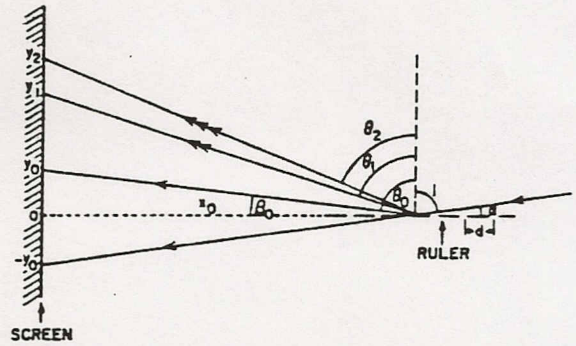

FIG. 2. Angles and distances for calculating wavelength from the diffraction pattern.

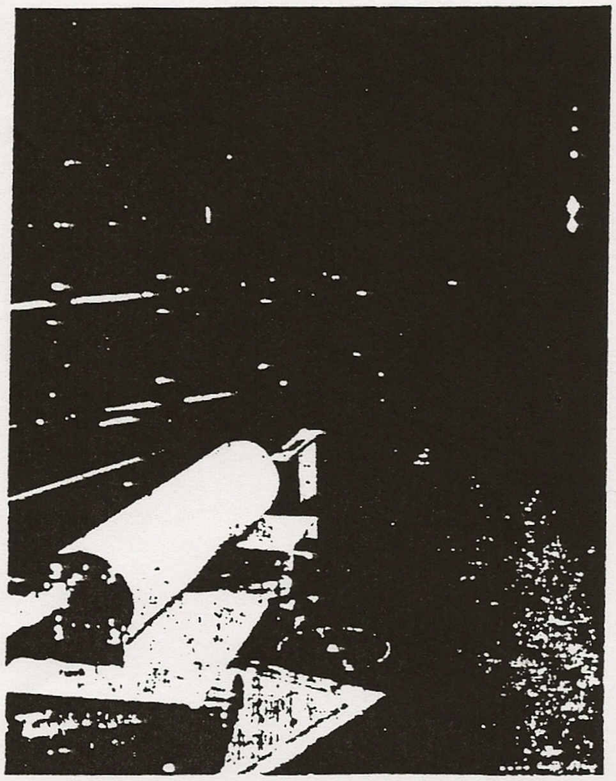

FIG. 1. Arrangement of the apparatus. The laser in the foreground shines on the rulings of a ruler, producing the diffraction pattern on the far wall.

It is also possible to demonstrate that the pattern is not clearly developed if the screen is too near the ruler, thus showing that the phenomenon is a case of Fraunhofer diffraction.

In a lecture-demonstration experiment, the distances x_0 and y_n were measured very hastily. Nevertheless, the differences of $y_{n+1}^2 - y_n^2$ all were constant to within two percent of their average value. With a little care, the wavelength of light could be measured to an accuracy about one percent, in a lecture experiment taking only a few minutes, and with all measurements clearly visible to the students. If desired, the theory and calculations can be given as an exercise.

ACKNOWLEDGMENTS

I wish to thank K. Sherwin for constructing the laser, and L. Snyder and R. Rosewood for assistance in setting up the demonstration and in taking the photograph. Optics Technology, Inc. kindly made available a prototype discharge tube of the kind to be used in their model 170 gas laser.