A tabletop experiment for the direct measurement of the speed of light

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We describe a tabletop experiment for the direct measurement of the speed of light to an accuracy of a few percent. The experiment is accessible to students not majoring in science or engineering. The experiment can include a measurement of the index of refraction of a sample. We discuss the experimental apparatus and safety considerations. The results and limitations of the experiment are analyzed, based partly on our experience in employing the experiment in student laboratories.

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I. INTRODUCTION

In this note we discuss the details of a tabletop experiment for directly measuring the speed of light in air. The experiment can be performed by undergraduate students, including nonscience or nonengineering majors and perhaps also by high school students. The principle of the experiment dates back at least a few hundred years to that of Galileo. We obtain the speed of light from the time of flight by measuring the time light takes to travel a certain distance and back. The experiment uses a pulsed laser and an oscilloscope to measure the time of flight. The optical path is in air so that the laser beam is visible, the experiment is safe, the result is reasonably accurate, and the experiment is inexpensive.

There are many experiments to measure the speed of light, including a number of interesting time of flight measurements.[1–9] However, we could not find any that have the same desirable features. In particular, most of the simpler experiments require a large space. Although it is possible to use a sinusoidally modulated laser, we chose a pulsed laser because a pulsed laser only emits light for a fraction of the time so that the average power of the beam can be an order of magnitude smaller, which is important for safety considerations. Also the fast rise time of a pulsed laser makes it easier to perform timing measurement, and the signal is more robust against distortions which can occur for a variety of reasons, such as detector saturation and an impedance mismatch. An experiment that is similar in spirit is discussed in Ref. 10.

II. EXPERIMENTAL APPARATUS

The experimental setup is simple and is shown in Figs. 1 and 2. A pulsed laser beam is reflected off a mirror and detected by a photodiode. An oscilloscope is used to obtain the time of flight $\tau=2L/c$, where L is the (one way) length of the path. The circuits for the pulsed laser and the detector are shown in Figs. 3 and 4. The pulsed laser has a quartz generated pulse modulation frequency of 1.0 MHz, a wavelength $\lambda=650$ nm, and an average optical output of 30 μW (30 pJ/pulse). We used a 200 MHz oscilloscope with 2 GHz sampling (Tektronix TDS2022). An example of the observed signal is shown in Fig. 5. All parts different from the base value (zero voltage in Fig. 5) can be regarded as parts of the signal. When the length of the optical path is changed, the signal profile is unchanged but it incurs a time delay. The experiment relies on measuring this delay, which can perhaps be most easily read off from the shift in the negative peak time. Let us explain the main characteristics of the signal: The prominent negative peak is the amplified photocurrent from the photodiode. The slow recovery due to the capacitance (200 pF)–resistor (68 Ω) circuit in the detector (Fig. 4) with a time constant 14 ns and some reflection signal can also be seen.

The total space required by the experimental equipment is about 1.5 m × 0.5 m. The signal shown in Fig. 5 was taken at a distance of around 1 m, but the strength of the signal is independent of distance for $L \lesssim 10$ m because the laser light is well collimated and is essentially limited by diffraction. The beam size is $D \simeq 7$ mm and should be easily detectable at 100 m and much further with focusing elements, especially given the signal to noise ratio evident in Fig. 5. The beam widening due to classical diffraction is $(\lambda/D) \times 100\ \mathrm{m} \approx 1$ cm. This small beam size can be useful in other projects using larger distances, as discussed in Sec. III. We have checked that the signal is easily picked up at $L=20$ m with no measurable loss in its strength without any additional focusing.

The average power of the laser light is 30 μW, and the measured pulse width is 2.5 ns. This pulse width is an upper bound because the high frequency measurement is limited by the performance of the oscilloscope. Although it is not advisable to observe the laser light without safety glasses, the laser beam does not seem to necessitate such safety precautions.[11] The laser circuit (see Fig. 3) is designed with a feedback circuit to keep the average light output constant. For safety reasons we would like the output to be small. However, we also would like the beam to be powerful enough to be visible even in a lighted room. The average output was chosen to satisfy both conditions.

We now briefly comment on the design of the pulsed laser, the detector, and the approximate cost of the experimental apparatus. The most expensive part of the apparatus is the oscilloscope (Tektronix 2022B). Such oscilloscopes are now standard and affordable, especially considering that they can be used for many purposes. Other less expensive 200 MHz

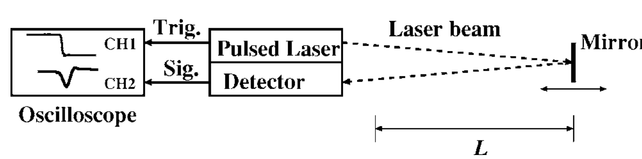

Fig. 1. A schematic of the experiment. A pulsed laser beam is reflected off a mirror and is picked up by the detector. The signal from the detector is measured using an oscilloscope, triggered by the outputs from the pulsed laser unit.

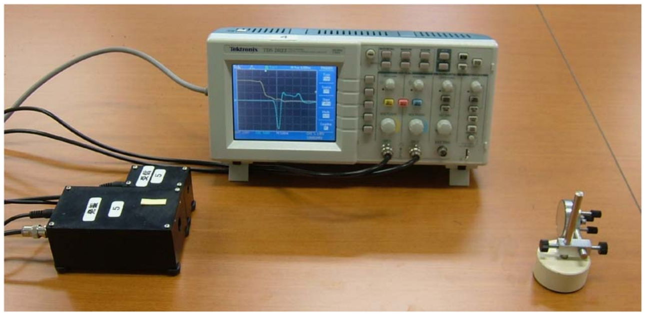

Fig. 2. The experimental apparatus with the oscilloscope (32 cm wide), the laser unit (12.5 cm long), the detector unit (6.5 cm long), and a mirror.

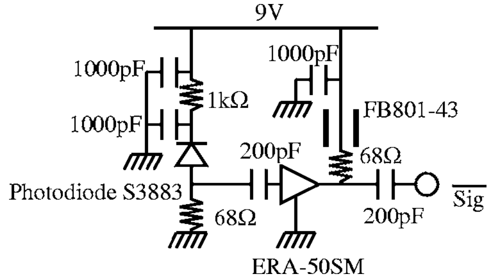

Fig. 4. Circuit diagram for the detector. The photocurrent from the photodiode is converted to a voltage by a resistor (68 Ω), which is amplified to obtain the signal.

oscilloscopes are available. We assembled the laser and the detector systems, and the parts should cost less than $300.

The output of the laser diode (DL-3147-260, Sanyo) depends on the ambient temperature, but is controlled by a feedback loop containing a generic operational amplifier (μpc842C, NEC). The beam is collimated by a lens (LT220P-B, Thorlabs) with good directional accuracy. Such accuracy, particularly in the vertical, is useful for setting up the experiment with ease. The 1 MHz clock module is generic; for example, a Kyocera KCJXO5-1.000C51C00 can be used. In the detector we adopted the S3883 photodiode (Hamamatsu Photonics) because of its performance and its relatively large sensing area. The photocurrent is amplified by an ERA-50SM (Mini-Circuits).

Although the laser and the detector units are inexpensive to build, their performance is high enough so that if we were to substitute them with a commercial pulsed laser and detector, the system would be more expensive by at least an order of magnitude. In particular, a pulsed laser system with a nanosecond time scale tends to be expensive. One commercial solution we could find is Advanced Laser Diode Systems PIL067 with EIG1000D ($8,340). However, safety issues need to be considered and a ND filter or safety glasses might be necessary. An example of a detector that should be adequate is Menlo Systems FPD310-FV ($1190 from Thorlabs), although for this particular purpose, it might not be as easy to use as the one we designed. Although we have not had the opportunity to test these pieces of equipment, they should be adaptable to this experiment judging from the specifications.

III. STUDENT PROJECTS

Students measure the difference τ in the time of flight at various values of L from the time measured at a reference point labeled $L=0$. (We use $L=0.2, 0.4, 0.6, 0.8, 1$ m from the reference point.) In particular, the students measure the temporal shift in the signal, τ, compared to the signal for the reference point. The reference point is located at an arbitrary point in the laser beam path as long as it is further from the light source and the detector by more than $\approx$30 cm for ease of alignment and to avoid errors due to the beam-detector separation. This error is around 0.3% at 30 cm. The students then plot the data points and fit a straight line to obtain the speed of light. By measuring the delay for various L, it is difficult for students to miss the meaning of the finiteness of the speed of light. Taking multiple measurements increases the accuracy and decreases the possibility of making simple mistakes in the procedure. We recommend not using lengths such as $L=1.5$ m, which leads to round numbers for τ (10.0 ns in this case). In our experience, using such lengths tends to mislead students into believing that the experiment has an unreasonably small error, because they usually obtain 3.0×10^8 m/s as a result.

The dominant source of error is the measurement of the time interval τ. With the 200 MHz oscilloscope that we use, the error in the time interval is $\Delta\tau \approx 0.2$ ns. With $L=1$ m and $\tau=6.7$ ns, the relative error in the velocity of light is 3%. The error can be reduced by using a larger value of L or a faster oscilloscope. A systematic error is introduced by not taking into account the lateral shift in the beam. Although a correc-

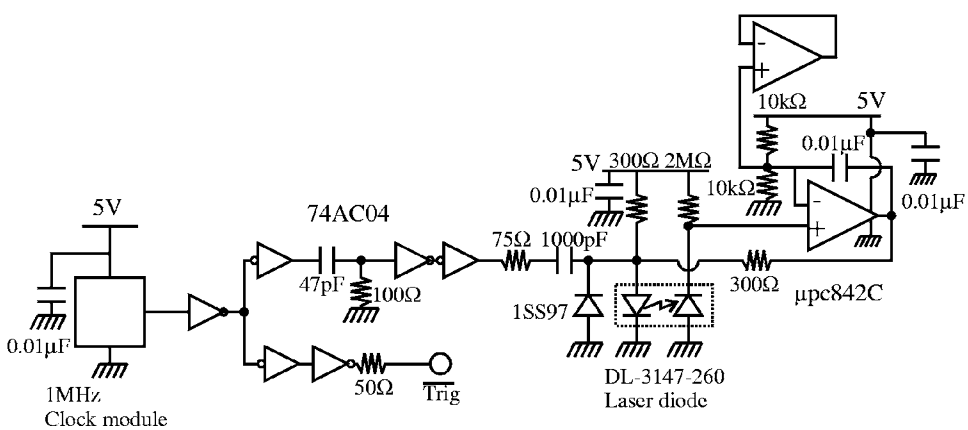

Fig. 3. Circuit diagram for the pulsed laser. The rectangular wave from the clock module is used as a trigger signal. The positive pulse derived from the wave is used to drive the pulsed laser emission. An operational amplifier is used to keep the laser power constant at 30 μW.

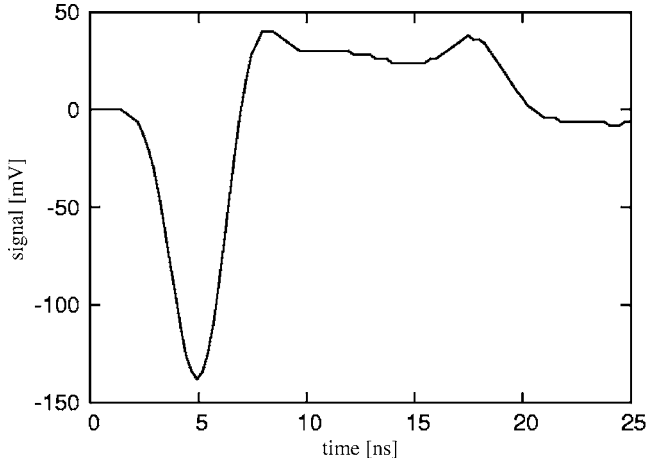

Fig. 5. An example of the detected signal (see the text for the explanation of the signal profile).

tion for this lateral shift can be incorporated, it is an order of magnitude smaller than this error, as explained above.

The result from the experiments performed by students at our university is $c_{\text{exp}}=3.04\times10^8$ m/s, with a standard deviation of 0.15×10^8 m/s (123 samples). This result is consistent with the speed of light and our previous error estimate of 3%. An example of an experimental result is shown in Fig. 6. Overall, we find the precision of this experiment to be satisfactory for our purposes.

We briefly discuss an extension of the experiment. By including some refractive material in the optical path, we can measure its index of refraction from the time delay, τ_{delay}, using the relation

$$c\tau_{\text{delay}} = (n-1)L_1. \tag{1}$$

Here, L_1 is the actual length of the refractive material in the optical path. We used an acrylic rod of $L_1=0.75$ m, which has an index of refraction $n=1.49$. The relative error in $n-1$ is 20% (for $L_1=0.75$ m), so that the absolute error in the index of refraction for our setup is 0.1. In our student experiments the average result is $n_{\text{exp}}=1.50$ with a standard deviation of 0.12 (71 samples), in agreement with our analysis.

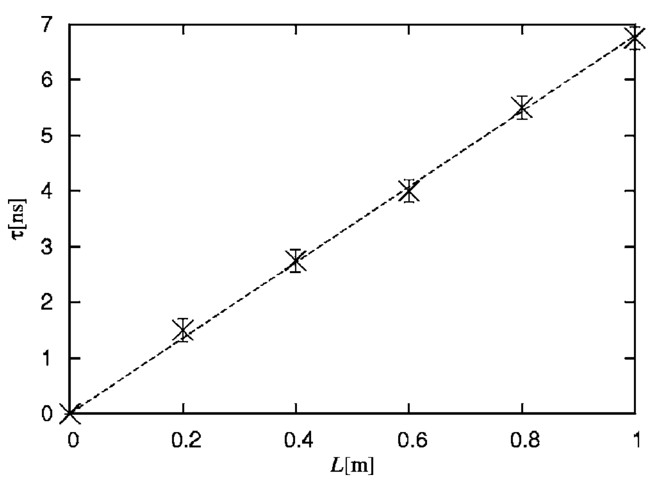

Fig. 6. Data from an experiment performed by students. In this case the fitted line (dashes) corresponds to $c_{\text{exp}}=2.94\times10^8$ m/s. The data at $L=0$ is the reference point and corresponds to $\tau=0$ by definition.

Because the light is attenuated by the refractive material and there are more pieces to align, it is more difficult to pick up the signal, but students can do so with a little effort. This part of the experiment is easier to perform without a mirror. The signal has no discernible difference from the signal without the dielectric shown in Fig. 5 except its strength, which is reduced by a factor of $\approx 1/4$ in our case. This signal loss stems mostly from the distortion caused by the inhomogeneity in the general purpose acrylic material we use.

The error in the speed of light measurement can easily be reduced to a percent or less if we use a path length of a few meters, instead of a meter. We can further increase the optical distance to obtain a more precise measurement of the light speed, by employing basically the same method. We can also measure the index of refraction of various materials. This measurement requires a larger optical length. Alternatively, we can use the experiment to measure distances using the speed of light as an input. This measurement can lead to an empirical understanding of how distances are measured in various areas, such as distances within the solar system and the GPS system.

IV. DISCUSSION

We have discussed a tabletop experiment to measure the speed of light to a few percent. To achieve such accuracy, the quality of the signal and the detector are crucial. We have done this experiment, including the index of refraction measurement, with roughly 600 students majoring in humanities and social sciences (mostly first-year) at Keio University. The experiments have been conducted with no problems and the results have been quite satisfactory. The experiment is done in groups of one or two students in a 3 h session, including the time for explanation by the lecturer and the time to write a short report on the experiment. The experimental output leads to a good understanding of the finiteness of speed of light, which is otherwise not easy to experience directly. Most students achieve the kind of experimental accuracy as designed. Students can really see and understand that the refractive material "slows light down." For most of the students, it is the first (and maybe the last) time for them to handle an oscilloscope and this aspect is probably the most difficult part of the experiment. Most students seem to understand and enjoy the visualization of the signal provided by the oscilloscope. Other than some instructions on the use of the oscilloscope, the experiment can be performed independently by students in most cases.

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Rotating Helices. This apparatus demonstrates the motor principle discovered by Michael Faraday in 1821. Originally the central pillar was set higher, allowing the downward-projecting points to make contact with mercury in the cups at the tops of the helices. The downward electric currents in the wire helices cause them to experience a torque due to the fringing magnetic field of the U-magnet. The device, dating from the middle of the 19th century, is on display at the Monroe Moosenick Medical and Science Museum at Transylvania University in Lexington, Kentucky. (Notes by Thomas B. Greenslade, Jr., Kenyon College)