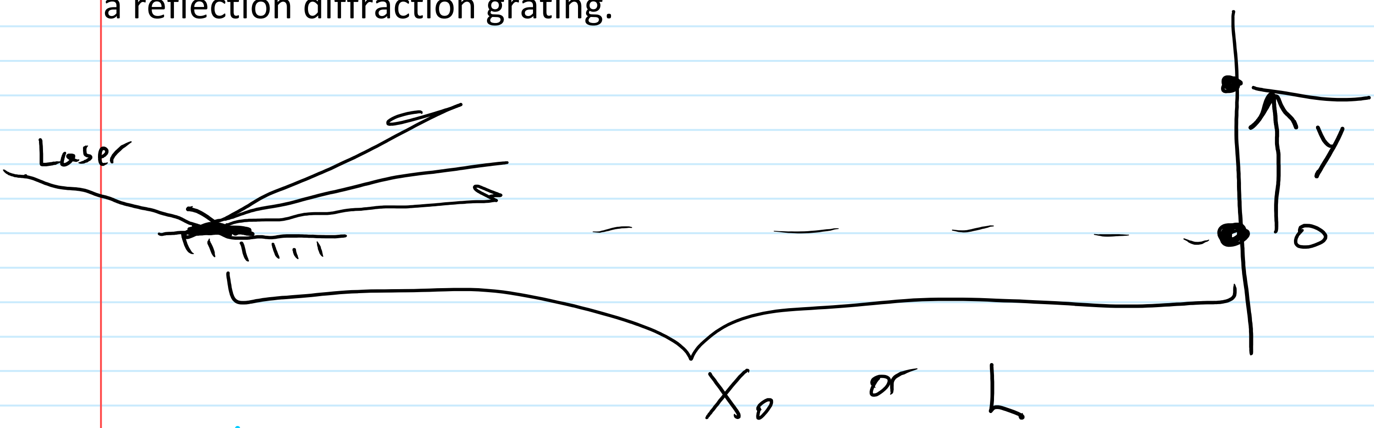
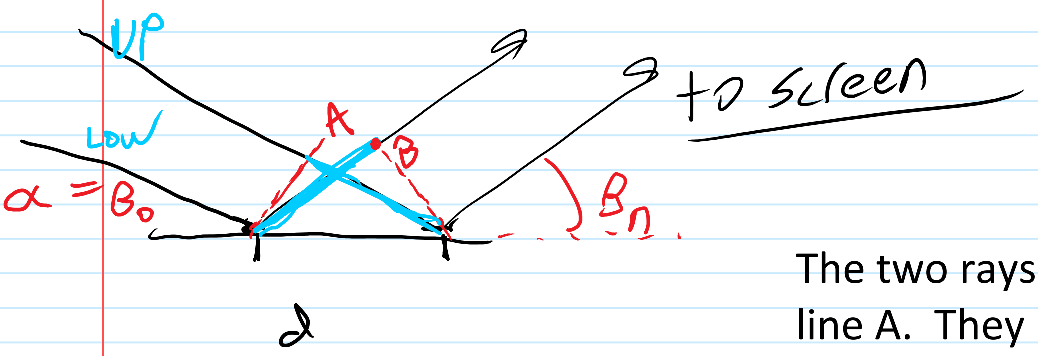
Arthur Schawlow is one of the Nobel prize winning fathers of Laser spectroscopy.

He developed what we are doing for lab as a demonstration. We are taking it to a bit higher precision and analysis. Schawlow used a ruler as a reflection diffraction grating.





The two rays are in phase at red line A. They now travel along different paths till red line B

The upper ray travels a distance ---blue line--farther than lower ray from A to surface. $=d\cos(\alpha)$.
Then ray lower travels an extra distance follow it's blue line$=d\cos(\beta_n)$ where the lower ray falls back.

After that, both rays travel the same distance to the screen.

The points hit on the ruler are scatter points from "between rulings" ---and as any small source, light is diffracted in all directions with some intensity depending on size.

The end result is IF the path difference = Whole number of wavelengths, then the waves add (at the endpoint-later--on screen) CONSTRUCTIVELY.

$$n\lambda = d\cos(\alpha) - d\cos(\beta_n)$$

In terms of Cartesian coordinates (which is what we measure) these trig functions can be expressed as:

$\alpha = \beta_0$

$$d\cos(\alpha) = \frac{x_0}{\sqrt{x_0^2 + y_0^2}}$$

$$d\cos(\beta_n) = \frac{x_0}{\sqrt{x_0^2 + y_n^2}}$$

We can do a few different things now--going back to our original path difference condition.

1) We can make a small angle approx which is the same as taking the first term in an expansion of the radical term for $y_n \ll x_o$ (which it might be). Then solve for y_n^2 vs. n, plot the resulting straight line, use slope to find wavelength.
2) OR we could solve for y_n^2 exactly---no approx---and now we have a more complicated function of $y_n^2(n)$ as a function of n. Whatever the function is--I can insert some of the measured constants and do a non-linear fit using tools like python, matlab, or origin. etc.

The difference in these---If I make a small angle approx, then I need to clip the data at an "n_{max}" small enough to ensure the approx is valid---uh---how valid?

Or if I make no approx, I can keep all the data---but I must learn how to enter a non-linear fit into some program.

Let's continue with some of the math so you see at least some terms.

$$\cos(\alpha) = \frac{x_0}{\left(x_0^2 + y_0^2\right)^{1/2}}$$

$$= \frac{\cancel{x_0}}{\cancel{x_0}\left(1 + y_n^2/x_0^2\right)^{1/2}}$$

$$\approx 1 - \frac{1}{2}\left(y_n^2/x_0^2\right)$$

Use your fav expansion series keep first term. Small y/x here.

OK--if you do the same with the other term, you'll get what Schawlow got.

$$y_n^2 = \frac{2\lambda x_0^2}{d} n + y_0^2$$

Which you can fit to a line, get the slope, and use your measured values to determine the wavelength.
BUT YOU MUST ANSWER THE QUESTION "HOW BIG AN n SHOULD I KEEP"?

If we want to do things exactly, then we can. We just need more computational power than Arthur Schawlow had available in the mid 60's at Stanford.

$$n\lambda = d\, x_0 \left(\frac{1}{\sqrt{x_0^2 + y_0^2}} - \frac{1}{\sqrt{x_0^2 + y_n^2}} \right)$$

$$= d \left(\underbrace{\frac{1}{\sqrt{1 + y_0^2/x_0^2}}}_{\text{this is a const we have}} - \frac{1}{\sqrt{1 + y_n^2/x_0^2}} \right)$$

$$n\lambda = d \left(\text{Const} - \frac{1}{\sqrt{1 + y_n^2/x_0^2}} \right)$$

I'll leave it to you to solve for y_n^2, which is pretty easy from here.

In order to fit your data
You will need to pick software,
enter your array of data,
and decide what parameters to fit.

n	y_n^2

Minimum---wavelength.
We have measurements for x_o and d.
Also y_o can be tricky.

At most we could fit all four parameters---but non-linear fitting programs require that we make pretty darn good guesses to do that.

In your HW on this---I've given you the task of comparing the numbers you get from the approximate model ---generate y_n^2 (approx) and then generate numbers using the exact model.

You MUST compare the results of the two models to determine out to what n_{max} you should keep in your dataset. Error bars are one indicator of this.

Make sure you do this so you know how to analyze your data---or else in the words of Mr. T----
https://www.youtube.com/watch?v=4FPGJrzzq_I