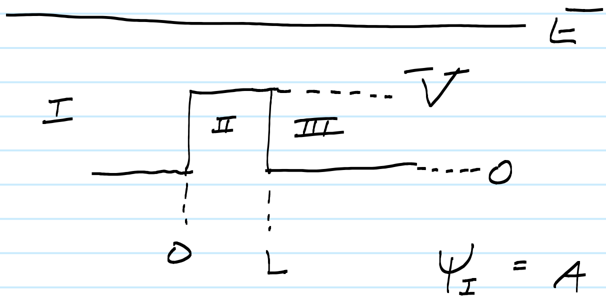
CH 15 Modern notes Phys 3300

15.1 Barrier —
note we are focusing on system results for several cases



$E > V$

Just like previous

$$\psi_I = A e^{iK_I x} + B e^{-iK_I x}$$

$$\psi_{II} = C e^{iK_{II} x} + D e^{-iK_{II} x}$$

$$\psi_{III} = F e^{iK_I x}$$

Like the well, we could consider sending in different energies (bound or free). Here, for the barrier---we can send in particles with energy over the well--or under the top of the well. With very few changes--we get mathematically similar results.

$$K_I = \frac{\sqrt{2mE}}{\hbar} \qquad K_{II} = \frac{\sqrt{2m(E-V)}}{\hbar}$$

Get the same expression as previous for well — for T

$$T = \left[1 + \frac{\left(V \sin K_{II} L\right)^2}{4E(E-V)} \right]^{-1}$$

Still $K_{II} L = n\pi$ gives $T = 1$

(If $T = 1$ then what is R)

If we consider the case with E< V   then things get weird (classically)

15.2 Tunneling

$E < V$

$$\psi_I = A e^{ik_I x} + B e^{-ik_I x}$$

$$\psi_{II} = \underbrace{C e^{\kappa x} + D e^{-\kappa x}}_{\text{note inside finite region keep both.}}$$

$$\psi_{III} = F e^{ik_I x}$$

OK--note that the energy and potential have come up in region II---to define a real number for kappa .   The solutions in region two are not travelling waves (left or right), BUT INSTEAD, are exponential growths or decays (we coulda used Sinh() and Cosh() ---and sometimes we do). We know the form of solutions for these potentials in each region.

Apply same old B.C. and then do lots of algebra

note basically $\pm i k_{II} \Leftrightarrow \kappa$

we expect very similar results but $\sin \longrightarrow \sinh$

$$T = \left[1 + \frac{\left(V \sinh(\kappa L)\right)^2}{4E(V-E)} \right]^{-1}$$

Note you must clearly write k's and κ etc

$$\kappa = \frac{\sqrt{2m(V-E)}}{\hbar}$$

Here $T < 1$ for any $2\delta L > 0$
Which it must be

Some gets reflected!

The takeaway here is that

- The particles don't have enough energy to get over the barrier.
- But some fraction of them (finite probability) GET THROUGH THE BARRIER
- There is no classical analog to this Quantum tunneling .
- For a continuous beam of particles---some of the particles are "in the wall".

Interlude: Other quantum systems----

"I must be allowed to speak". https://www.youtube.com/watch?v=9357t5Q79kg

OK--

- there are both barriers and wells that can be formulated in two or three dimensions (hard spheres for example).
- Coulomb potentials--like Hydrogen
- Other interactions yet to be described
- The Harmonic Oscillator (coming) 1 dim
- The Harmonic Oscillator 3 dim
- Different interactions can be solved numerically
- We have looked at some set-ups and some solutions
    - In most cases we skipped the derivations

The Harmonic Oscillator (1 dim)

15.3 Harm OSC

Parabolic Potential

— one dim

$$V(x) = \frac{1}{2} K x^2$$

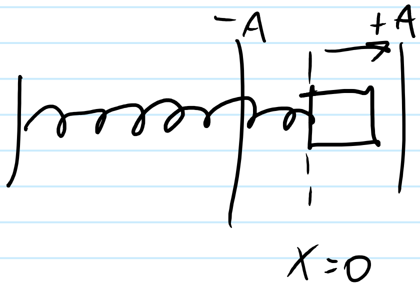
spring const here

— try a different way to write

$$= \frac{1}{2} m \omega_c^2 x^2$$

$$\omega_c = \sqrt{\frac{K}{m}}$$ Classical

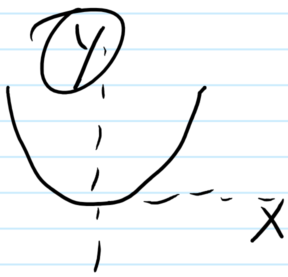
Not from E_n sol to S.E.



Remember this is a mass on a spring
The potential is $(1/2)kx^2$....
but there is no "parabola" physically

It is a one dimensional vibrational system.

Not



The classical frequency ω_c above---is not the same as the quantum frequencies of an nth state----- $\omega_n = E_n/(\hbar)$ ......The classical number is just carrying constants related to the system.

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi_n}{dx^2} + \frac{1}{2} \underbrace{m \omega_c^2}\, x^2 \psi_n = E_n \psi_n$$

mass and spring const stuff — the system

⋮

Many pages of solution

⋮

$$E_n = (n + \tfrac{1}{2}) \hbar \omega_c = (n + \tfrac{1}{2}) h \nu_c$$

Equally spaced energy levels

& any bound system can be approx

& any bound system can be approx
S. H. O.

Note: I have not given you the wavefunctions, but only the energies of the system.

The wavefunctions have the typical appearance inside the potential well-----0) one half cycle--with a little leaking out the edges at each side, 1) two half cycles---with a little……. 2) three half cycles…..with a little …….4)

The stationary state wavefunctions (solutions for ψ_n)---like all of our solutions--form an ORTHONORMAL BASIS SET.
(THEY HAVE A PROPERTY LIKE--"PERPENDICULAR" UNIT VECTORS)

Note ω_c or ν_c is not the ψ_n Matter wave frequency —

$$\omega_n = E_n/\hbar = (n+\tfrac{1}{2})\omega_c$$

The classical oscillator has one frequency and allows for Any energy.

The classical probabilities are high at the turning points and low in center where it moves fastest.

— A good problem would be to plot the classical probability density.

The classical system---the probability in a region is proportional to the relative amount of time spent in a given region, which depends on "how fast" and the period.

The minimum energy of Quantum S.H.O is not zero but is $\frac{1}{2}\hbar\omega_c$

— So what ⟶

the lowest state still has KE.

— We must measure - both - classical & quantum from the bottom of the well rather than from the potential at far far away

There are no quantum states with zero energy.

Note in example 15.4

$\psi_1 \propto x\, e^{-\alpha x^2/2}$ find α

That is of form

$$f(x)\, e^{-\alpha x^2/2}$$

$f(x)$ is x here, then must normalize.

https://www.youtube.com/watch?v=671AgW9xSiA