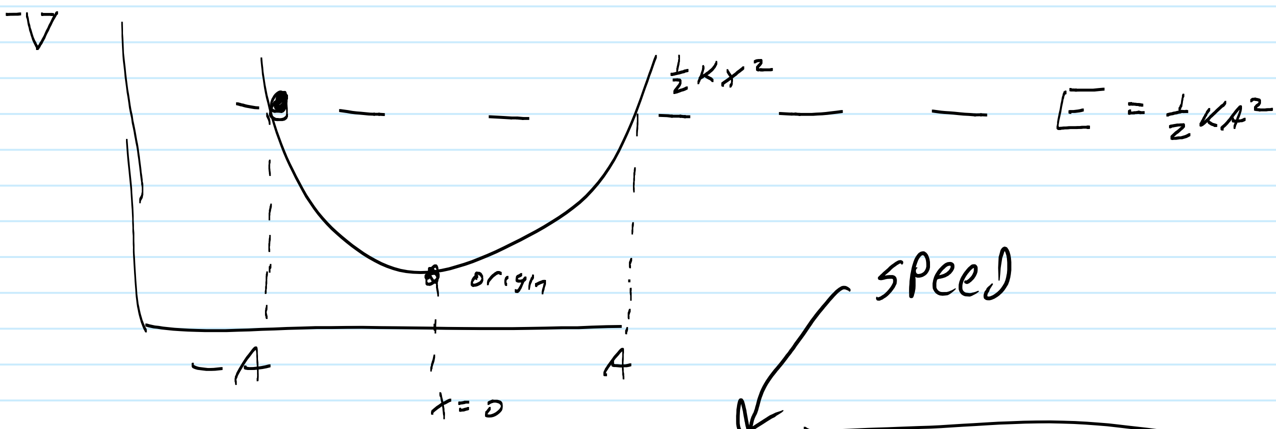
A comparison of quantum and classical results.
We have discussed some of the quantum 1 dimensional potentials (square well, harm osc, coulumb like).

For the harmonic oscillator potential it is good to compare and contrast our classical results with quantum results (you will probably do a full derivation of quantum harmonic oscillator solutions in your quantum course).



speed

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 \qquad v = \sqrt{\frac{2E}{m} - \frac{kx^2}{m}}$$

$$\omega_c = \sqrt{\frac{k}{m}} \qquad = \sqrt{\frac{kA^2}{m} - \frac{k}{m}x^2}$$

Classical frequency

$$= \omega_c\sqrt{A^2 - x^2}$$

In the quantum harmonic oscillator--the same angular frequency ω_c is used to carry around the information regarding k and m---but it is just a parameter.

In the classical system--for a given "spring" with "k" and mass "m". There is only one frequency of oscillation for the system. ω_c

We know the motion of the system classically. In order to describe the probability distribution (as is done in quantum mechanical systems)---we need to describe how much time the classical particle spends in each region. --then compare to the total time period

$$X = A\cos\omega_c t$$
$$V = -A\omega_c \sin\omega_c t$$

Time spent in finite region ΔX is

$$\int \left(dX / |V| \right) \quad * \quad \text{Compare to } T/2$$

$$\int \frac{dx}{\omega_c\sqrt{A^2-X^2}} = -\frac{1}{\omega_c} \arctan\left(\frac{X\sqrt{A^2-X^2}}{X^2-A^2} \right)$$

If you evaluate this from x=-A to x=+A (half cycle) then you will get the time it takes to go from -A to +A ......if you now divide by T/2 ---you will get "1" for the probability of being spending time between -A and A — can eval any range

The probability of being "At x" ---meaning from x to x +dx is simply

$$P(X)dX = \frac{dX}{|V|} * \frac{1}{(T/2)}$$

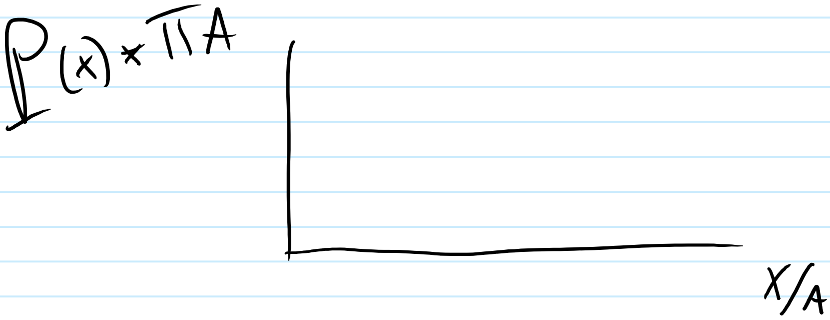
So the probability density is just

$$P(x) = \frac{2}{T}\,\frac{1}{\omega_c\sqrt{A^2 - x^2}}$$

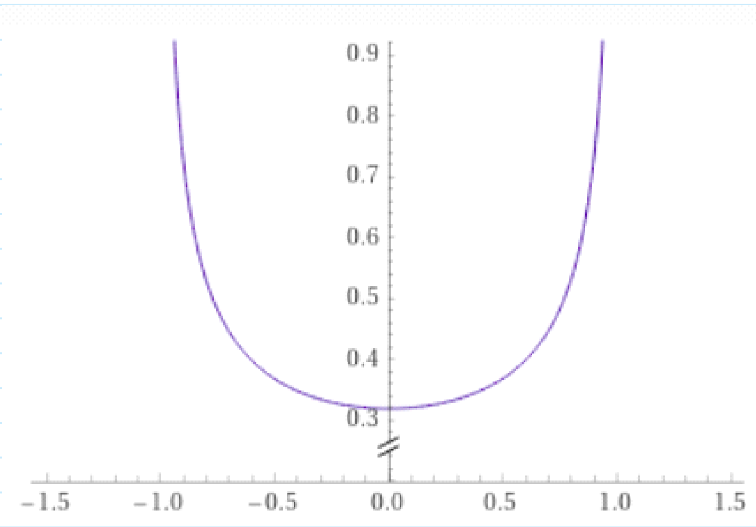
$$= \frac{2}{T\omega_0 A}\,\frac{1}{\sqrt{1 - (x^2/A^2)}}$$

$$= \frac{2}{\cancel{T}\left(\frac{2\pi}{\cancel{T}}\right)A}\,\frac{1}{\sqrt{}}$$

$$= \frac{1}{\pi A}\,\frac{1}{\sqrt{1 - (x/A)^2}}$$



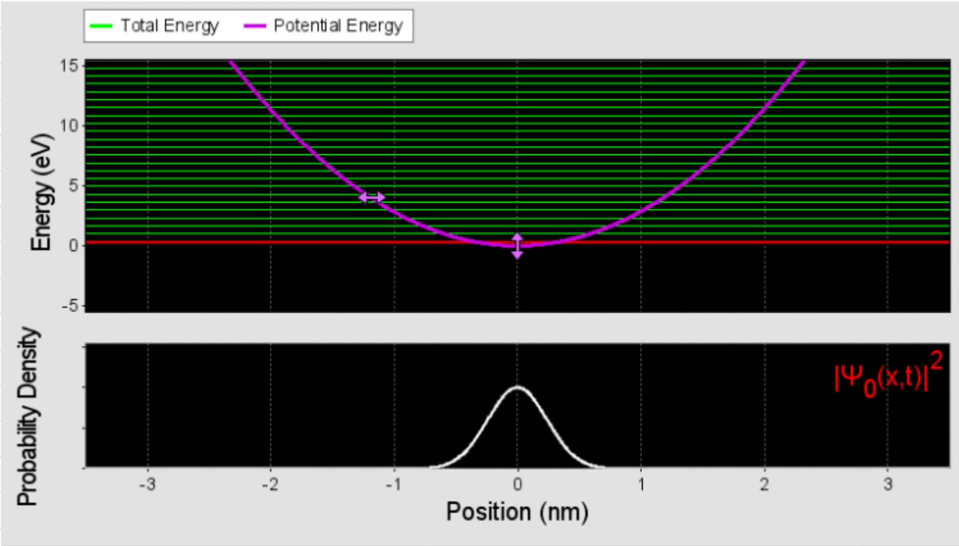
dimensionless plot



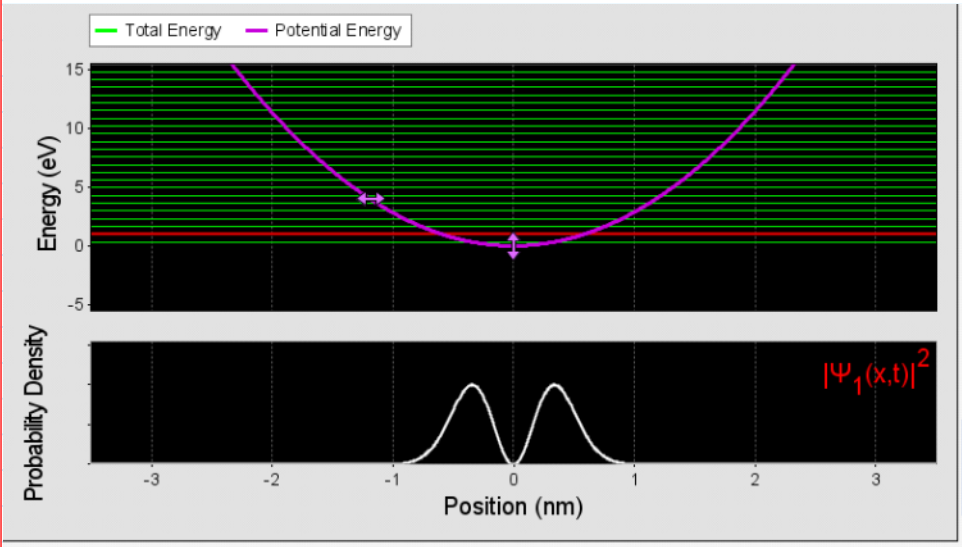
This is what we expect for the classical system

The probability is high at the turning points (the edges where the particle moves slowly). The probability is low where the particle is moving fastest---the center.

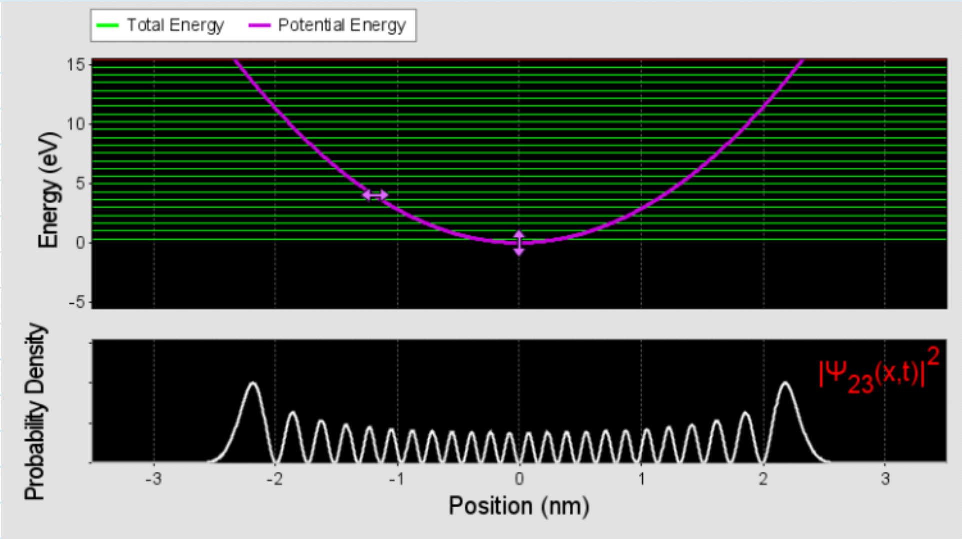
How does this compare to quantum---take some screenshots.



For the ground state--quantum---the probability distribution peaks at the center--where the particle moves fastest????---yup.



For state n=2---the center now has zero probability density---particle motion toward edges, but not yet classical like.



For state 23---we could draw an average line through this probability distribution (center of the up down motion)---and now we get something closer to the classical distribution.

For even higher state ---say a billion or more....the peaks would run together, I would not be able to discriminate between one peak and the next----(measuring tool precision)....and we would see the classical distribution.