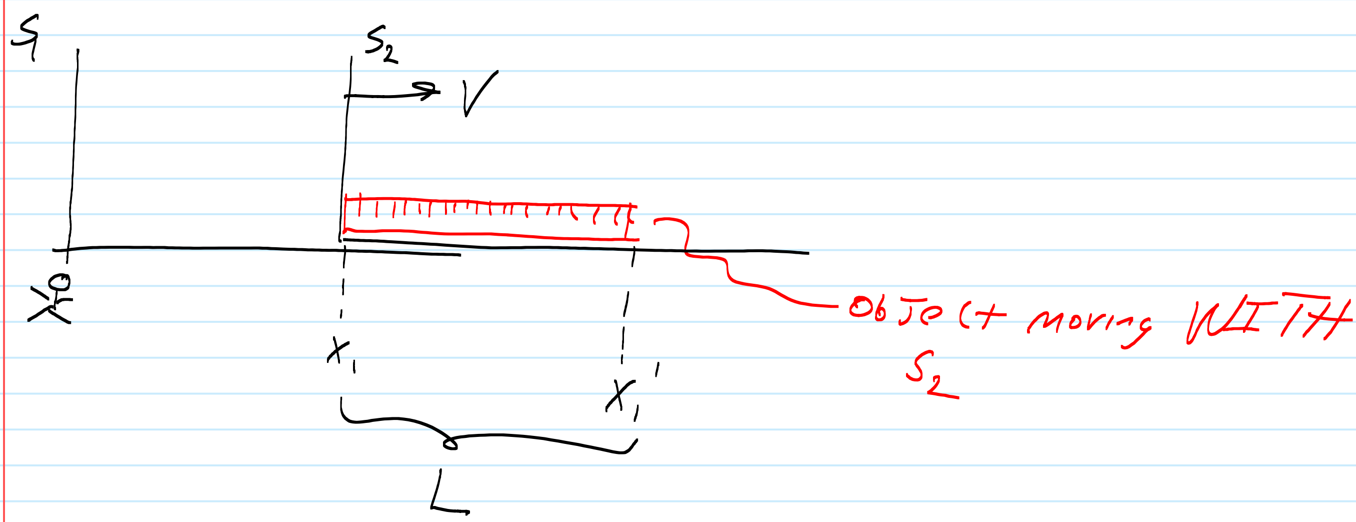
Ch. 4---Length, Time,Velocity --transformations

We are going to place an object in frame 2 (meaning the object moves with frame 2) and see what it looks like as observed from frame 1.



We measure x_1 and x_1' at the same time
somehow → $t_1 = t_1'$ (flash and record ends)

The requirement of simultaneity has always been there for a length measurement (for a moving object).

To get the length in frame 2 (the frame at which the meterstick is at rest) we measure x_2 and x'_2 (the prime indicating the far end--final). To get the length in frame 1 we measure x_1 and x'_1.

In frame 2 we can make measurements at any time since, that is the rest frame.

$$x_1' - x_1 = L$$ now do Lorentz transforms

$$x_2' = \gamma(x_1' - vt_1')$$

$$x_2 = \gamma(x_1 - vt_1)$$

$$x_2' - x_2 = \gamma(x_1' - x_1)$$

$$\underbrace{L_0}_{\text{at rest in } S_2} = \gamma L$$

$$L = L_0/\gamma$$

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}}$$ always > 1

Length contraction. ----moving objects are shorter in the dimension parallel to the motion.

This is not "appears shorter"---this is ---in every measurable sense, the objects dimensions are shorter.

The times corresponding to t_1 and t'_1 ---in frame 2 are irrelevant, since it does not matter when the measurements are made in the rest frame.

Same

Note $t_2 = \gamma(t_1 - \beta/c\, x_1)$

and $t_2' = \gamma(t_1' - \beta/c\, x_1')$

So $t_2 \neq t_2'$ $\rightarrow$ NOT simultaneous

The cube Example 4.1

Comments:

So you have a Borg cube (anyone---????) moving 0.800c in a direction parallel to one of the sides.

- The cube experiences length contraction in that direction
- The shape remains a cube
- As an observer, the cube does not appear as a cube to me---due to transit time effects
    - It takes longer for light to reach me from the far edge than from the near edge.
    - Imagine you are at a rock concert (Formerly the Grateful Dead)---and you are near one speaker and far from another---it will take more time for the same "note" to reach you from the far speaker than the near one. The time delay depends on size, and distances.

Watch the cubes movie clip

https://www.spacetimetravel.org/tompkins/tompkins.html

There is no spoon

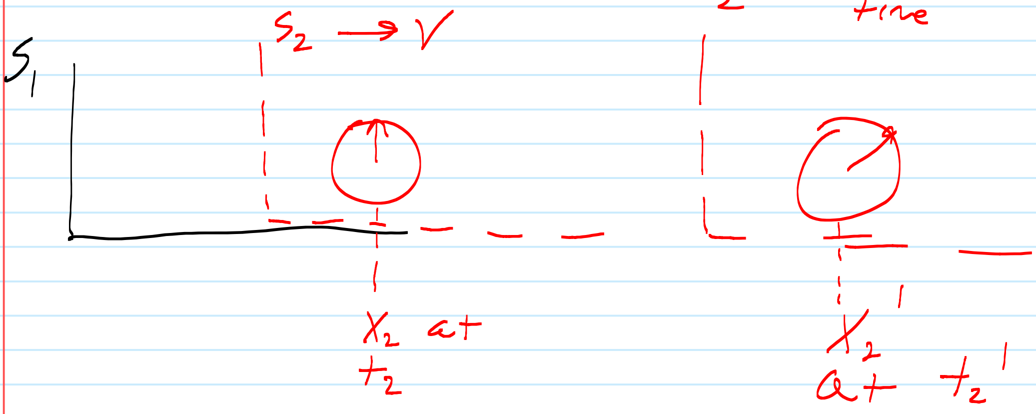
https://www.youtube.com/watch?v=uAXtO5dMqEI

What do I mean by this---"there is no spoon"---well-you never really have an accurate picture of any object. Light reaching you comes from different parts, experiencing different contractions, with reflected light that is differently Doppler shifted----there is no "true" representation of the object that you receive---We always see a manipulated image.

Time Dilation----A clock is stationary in frame 2 (which is a rocketship) You--the starship captain --observe the clock to behave normally.

What do I (in frame 1) observe?

Time DiLation



The clock is not moving inside your starship---so x_2 and x'_2 are the same. t_2-t'_2 define the proper time--the rest time interval in the frame of the clock.

So I am watching your clock---one tick has passed by----the question is, while this event happens (I'm watching the same event as you---your clock tick) ----how many ticks pass by on my clock?

$$t_1' = \gamma\left(t_2' + \frac{\beta}{c} x_2'\right) \qquad t_1 = \gamma\left(t_2 + \frac{\beta}{c} x_2\right)$$

$$T = t_1' - t_1 = \gamma\left(t_2' - t_2\right)$$

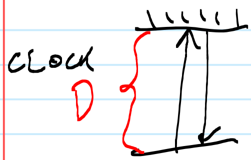
$$T = \gamma T_0$$

The x_2 terms cancel out---so I observe on my watch--many ticks go by (γ>1) on my watch--as I see your watch tick once. The time is greater on my watch---I see time running slowly for you ---each step you take, every motion, every event---you are running slow. (taking more time than normal).

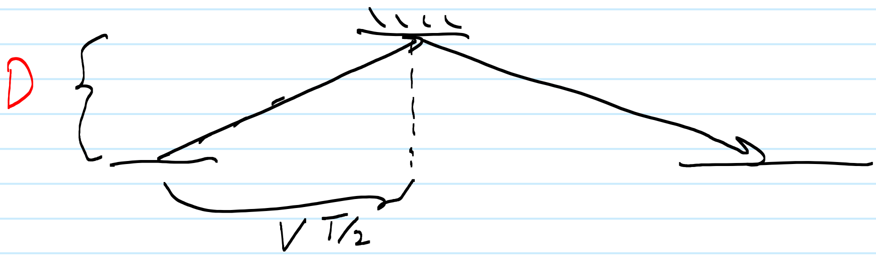
It is possible to derive time dilation directly and geometrically from pictures. And this is very convincing regarding the effect.

We consider observing a light clock---You are at rest with the clock, and you watch the emission of light from a diode travel up to a mirror and back to a detector. You measure the time.

I am watching the same clock--the same photons--travelling at the speed of light---but I see you in your rocket ship moving. The round trip of the photons (travelling the speed of light) is bigger to me--so it takes more time. Mind--we are watching the same event, same photons, etc.....but you use your watch for timing the event. I'll use mine.



at rest $\qquad 2D = cT_0$



do the geometry.

$$2\sqrt{D^2 + \left(\frac{vT}{2}\right)^2} = cT$$

$$4\left(D^2 + \frac{v^2T^2}{2^2}\right) = c^2T^2$$

$$4D^2 + v^2T^2 = c^2T^2$$

$$\underbrace{4D^2} = (c^2 - v^2)T^2$$

$$(cT_0)^2 = (c^2 - v^2)T^2$$

$$T_0^2 = \frac{c^2 - v^2}{c^2}\,T^2$$

$$T_0 = \underbrace{\sqrt{1 - v^2/c^2}}_{1/\gamma}\;T$$

When you are moving near the speed of light--I observe you are thinner--and your watch runs slowly.

Also several other distortions due to extent, and transit time effects.

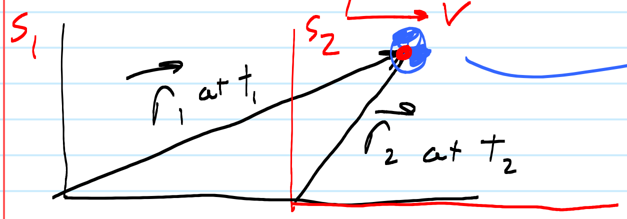
Now that we understand what happens to length and time---for a meterstick sitting still in your reference frame (you move with respect to me) and a clock sitting still in your reference frame---we must ask the question, what if you throw a ball?

If the object we are observing has a velocity in your reference frame (frame 2) then what is the velocity I observe in my reference frame?

The transforms involve each coordinate differently since distance (length) is impacted only in the x direction, but time is changed regardless of the motion.

We need to carefully apply our definitions of "VELOCITY" in each frame to derive the transformations.

Velocity transforms.



and the object moves

how does $\vec{V}_1$ relate to $\vec{V}_2$

Note: $\vec{V}_1$ has 3 comp V_{1x}, V_{1y}, V_{1z}

$\vec{V}_2$ " " " V_{2x} V_{2y} V_{2z}

We also have "v" ---the relative motion of ref frame.

The math

$$x_2 = \gamma(x_1 - v t_1) \qquad t_2 = \gamma\left(t_1 - \frac{\beta}{c} x_1\right)$$

We need to consider things like $v_{1x} = dx_2/dt_2$

Recall that:

Consider • to move incrementally
& recall differential calc

for $f(x,t)$ $\quad df = \frac{\partial f}{\partial x} dx + \frac{\partial f}{\partial t} dt$

we have $x_2(x_1, t_1)$
& $t_2(x_1, t_1)$

We have for dx_2 and dt_2

So

$$dx_2 = \gamma(dx_1 - v\,dt_1)$$

and $dy_2 = dy_1 \qquad dz_2 = dz_1$

Likewise time

$$dt_2 = \gamma\left(dt_1 - \frac{\beta}{c} dx_1\right)$$

From here we need to merely divide and simplify.

$$V_{2x} = \frac{V_{1x} - V}{1 - \frac{\beta}{c} V_{1x}}$$

$$V_{2y} = \frac{V_{1y}}{\gamma\left(1 - \frac{\beta}{c} V_{1x}\right)}$$

note x

Two effects – one due to x changing other due to time changing

$$V_{2z} = \frac{V_{1z}}{\gamma\left(1 - \frac{\beta}{c} V_{1x}\right)}$$

For the velocity transforms---

- It is not possible to get a transformed speed greater than c
- The observed velocities do not depend on time or position
- The transforms depend on the relative motion "v" and the initial velocity components.
- Reverse transforms--change labels and v to -v.
- Note example 4.7--angle for velocity may change
- Like regular kinematics--angle for position, and angle for velocity are not the same things.

Where are we---we have gone as far as we can with basics, and now must consider --more than simply "kinematics"--but must ask, what are the causes for various types of motion.

We already have preconceived notions about things like

- Momentum
- Force
- Kinetic and Potential Energy
- Mass
- etc---we may need to change our point of view.