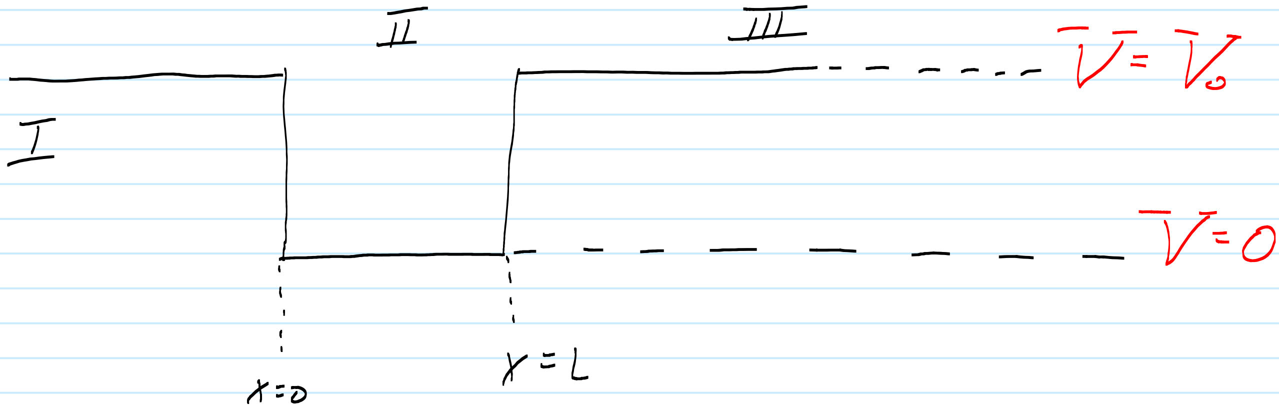
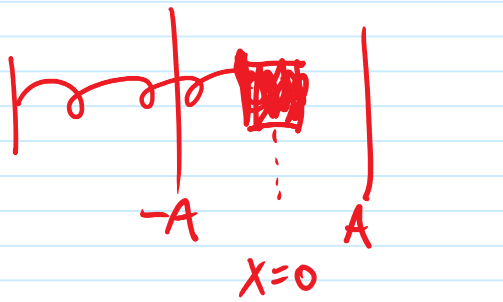
One dim wells — Not Hydrogen
Learning to work with S.E.



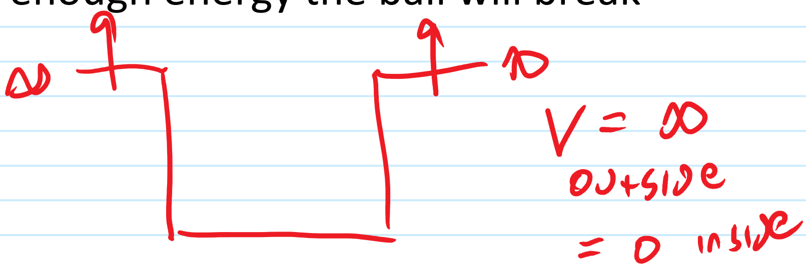
We could describe the well differently — shift up/down
left to x = −L/2 etc

What does this picture mean? If we had
A mass on a spring, in one dim, then
the potential would look like a parabola
(1/2 k x^2)



For our well--we have a one dimensional superball bouncing
between walls at 0 and L (or -L/2 and L/2)---we could make
dimensions anything. The walls may be infinitely hard (i), or if
the ball is thrown with enough energy the ball will break
through (ii)

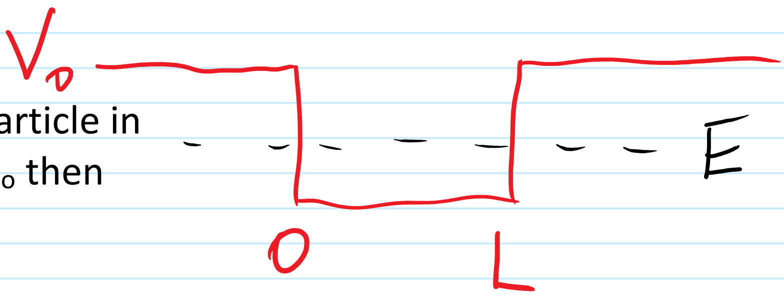
i



ii or

There are other potentials we might consider, but square wells (infinite or finite) are starting points. Hydrogen has a coulomb potential.

Square well
If the energy of the particle in the well is less than V_o then the particle is bound.



Inside the well   E-[V=0 in the well]=KE
Inside the well, KE is positive.

If a "Particle" has KE > V_0 then it is free.

Three regions — to left of well I
inside well II
to Right III

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi(x) + V(x)\psi = E\psi$$

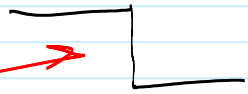
$V(x)$ is given by picture

B.C. at Left & right edges

$$\psi_I\Big|_{x=\text{left}} = \psi_{II}\Big|_{x=\text{left}} \qquad \text{also} \qquad \psi_{II}\Big|_{\text{right}} = \psi_{III}\Big|_{\text{Right}}$$

The S.W.E (time indep) describes the time independent wave function. Our interpretation is that $\psi^*\psi\, dx$ describes the probability of the particle being in this range dx---this must physically be smoothly continuous. A discontinuity in this---would be like --jump in velocity (infinite acceleration---bad).

So boundary conditions at left and right edge for BOTH THE WAVEFUNCTION AND FIRST DERIVATIVE.

If ψ has discontinuity 

Not √

then $\frac{d\psi}{dx} = \pm\infty$ which is proportional to momentum.

we require no discontinuities — and finiteness

and $\left.\frac{d\psi}{dx}\right|_{\mathrm{I}\ \text{left}} = \left.\frac{d\psi}{dx}\right|_{\mathrm{II}\ \text{left}}$ and II, III on right

again prop to momentum — so no infinite forces
no 2nd derivatives being infinite!

We have a differential equation, we have a set of boundary conditions ----and also NORM. (NORMALIZATION--NOT CHEERS).

We start by looking at the special case (diffeq -101)
THE INFINITE SQUARE WELL.
This just means the walls are really really rigid.

- We must solve in region I
- Region II
- Region III
- Apply boundary conditions
- Normalize
- Find any relevant constants (if needed).

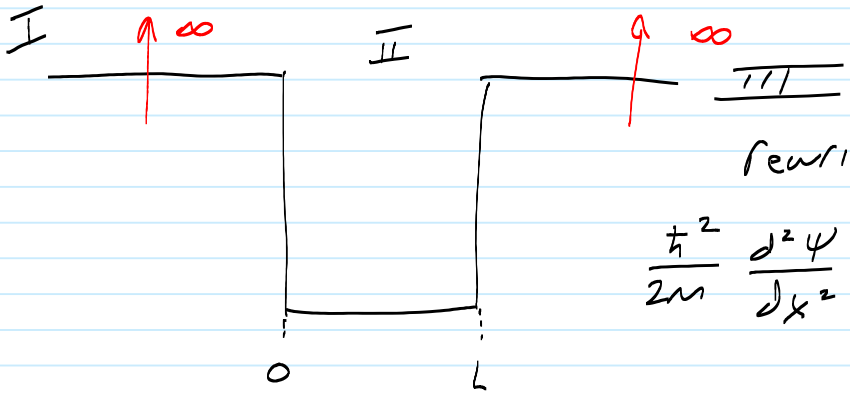
A Special Case — Infinite square well

$$\bar{V}_{x=-\infty \text{ to } 0} = \infty \qquad \bar{V}_{x=0 \text{ to } L} = 0$$

$$\bar{V}_{L\to\infty} = \infty$$

Note--these are not three different "V"s, but simply a single function that is piecewise continuous. V has a value everywhere.

This particle can never be free



rewrite S.E.

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -(E-\bar{V})\psi$$
$$= (\bar{V}-E)\psi$$

The interesting region is region II (the particle never makes it outside the well ---so the wavefunction ψ=0 in region I or region III---we show later).

Region II

write out S.E. in region II

(we know $\psi = 0$ in region I or III)

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = -E\psi$$

or more familiar

$$\frac{d^2\psi}{dx^2} + \left(\frac{2m}{\hbar^2}E\right)\psi = 0 \qquad \text{call } k^2 = \frac{2mE}{\hbar^2}$$

solutions are of the type

$$\psi_{II}(x) = A\cos(kx) + B\sin(kx)$$

could use $e^{\pm ikx}$

use $e^{\pm ikx}$

Diffeq 101

$\psi'' + k^2 \psi = 0$ Has solutions that look like $A\cos(kx) + B\sin(kx)$

diff A, B or $Ae^{ikx} + Be^{-ikx}$

or $A\cos(kx + B)$

These are all different ways of saying the same functions----we can put pieces together to make them look the same. We use the first, but all have constants A and constants B and also k.

Recall $k^2 = 2mE/\hbar^2$

Plug in the solution to check.

- When we normally solve diffequ---we have "k" given (like giving energy here)
- We have B.C. given
- We solve using "k" and B.C. to find constants A, B.

- Here we will find that there are only certain values of k that are allowed
- k_n ---that relate to specific energies E_n
  - Other values cannot ever satisfy B.C.
  - The energy values E_n are called the "eigenvalues" (of energy)
  - The relating wave functions are the "eigenstates" (standing wave or stationary states)
- With specific B.C and initial conditions --we can find everything.

Remember we are solving in region II. For a given eigen-number--n, we might have something like........

might call this ψ_{II_n}

$$\psi_{II_n} = A_n \cos(k_n x) + B_n \sin(k_n x)$$

and $\Psi_{II} = \sum \psi_{II_n} e^{-i\frac{E_n t}{\hbar}}$ to fit B.C.

In the last step, we put back together the full time dependent solution. (after finding the individual stationary states).

With $E/\hbar = \omega$

our solutions can be made to look like $e^{\pm i(kx \mp \omega t)}$

Waves

Where are we right now---we have the form of the solution for region II only.

$$\text{II} \qquad \psi_n(x) = A_n \cos(k_n x) + B_n \sin(k_n x)$$

We do not have --region I or III, we do not have the allowed k_n
A_n B_n ----

Begin by tackling region I and III---I told you that ψ is zero, but let's show it.

Now in regions I or III $V(x) = \infty$

Claim is that $\psi_I = \psi_{III} = 0$ — show

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (\infty)\psi$$

Keep the ∞ for now

$$\frac{d^2\psi}{dx^2} = \left(\frac{\infty}{\hbar^2/2m}\right)\psi$$

really $\infty - E$

$V - E$

Consider side I

solutions $\psi = Ce^{-\sqrt{(\ )}(x)} + De^{+\sqrt{(\ )}(x)}$

When x is $-$ the C term blows up

So to satisfy B.C. $C = 0$

for an ∞ well $e^{+\infty(x)}$ with x is neg

goes to 0 $\longrightarrow \psi_I = 0$

What boundary condition did we satisfy at negative x space----?
We can't have infinite wavefunction. No infinite probability .
We also believe that the particle is bounded to the universe, so the wavefunction must decrease as x goes out to infinity (either way). To satisfy BC---we make C in region I =0.

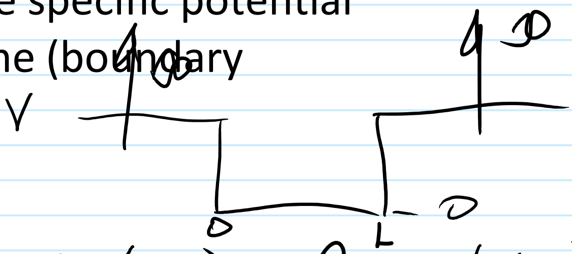
The term with D in it, goes to zero all on it's own---regardless of what finite value of D we have. exp(infinite* negative x val).

Likewise in region III $\psi_{III} = 0$

impose additional B.C.

So---ψ in regions I and III =0. We can focus on finding remaining constants in region II

We are given "m" and "L" and the specific potential (everywhere), and rules of the game (boundary conditions).



Back to region II $$\psi_n = A_n \cos(k_n x) + B_n \sin(k_n x)$$

We will start imposing boundary conditions ---first at x=0,

At $x=0$ $$\psi_I(0^-) = \psi_{II}(0^+)$$

$$0 = A_n \cos(k_n 0) + \cancel{B_n \sin(k_n 0)}$$

for all n possible $\rightarrow$ $$0 = A_n \cos(k_n 0)$$
$$\therefore A_n = 0$$

- Normally we would now impose B.C. $\left.\frac{\partial \psi_I}{\partial x}\right|_{x\to 0^-} = \left.\frac{\partial \psi_{II}}{\partial x}\right|_{x\to 0^+}$
- Since ψ_I was zero---
  - Extra constants got knocked out from
  - zero on the left---no need for doing the work from this extra B.C. here
  -
  -
- Onto x=L where same deal occurs (no need for derivative continuity----there is a known kink in this wavefunction at infinite barrier---goofy and unreal, but we chose that idealized problem).

B.C. at $x=L$

$$\psi_{III}(L) = \psi_{II}(L)$$

$$\star \quad 0 = B_n \sin(k_n L)$$

This is the eigenvalue part $\rightarrow$ find k_ns (if any) that make $\star$ true

Recall $k^2 = 2mE/\hbar^2$

$$k_n = \frac{n\pi}{L}$$

this will determine the energies — for allowed ns

The last constant is found in this case using Normalization.
Summary of events------

- In regions I and III we have $\psi(x) = 0$ ---BC---required for infinite potential
- In region II we imposed BC---that wavefunction is continuous at x= 0---gave A's=0
- In region II we imposed BC---that wavefunction is continuous at x=L, this gave allowed k_ns which gives Energy
- That leaves B's---which will be found using normalization.
- Think of each region being bounded by two sides with ψ ψ' being continuous
- At + and - infinity-----the wavefunction must not blow up (probably should be zero).
- The B.C. and Norm---give sufficient number of equations to solve for constants. (sometimes there is redundancy)

Normalization

$$\int_{\text{All space}} \psi^* \psi \, dx = 1$$

$$\cancel{\int_{\text{region I}} \psi_I^* \psi_I \, dx} + \int_{\text{region II}} \psi_{II}^* \psi_{II} \, dx + \cancel{\int_{\text{region III}} \psi_{III}^* \psi_{III} \, dx} = 1$$

$$\int_0^L B_n^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\longrightarrow B_n = \sqrt{\frac{2}{L}} \quad \text{for all } n$$

We have the A's, the B's, and the k --or n---or E for this system that allow for boundary conditions to be satisfied.

In other words---since we have solved the form of differential equation for different possible values of n---we have really solved an infinite number of diffeq---with differing values of the sep constant E

Putting it all together, and recalling that our solution is a piecewise continuous function.

$$\psi = \begin{cases} \psi_I = 0 & \text{for } x<0 \\ \psi_{II} = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x & 0 \le x \le L \\ \psi_{III} = 0 & x > L \end{cases}$$

recall diffeq

in Region II

$$-\frac{\hbar^2}{2m}\frac{\partial^2 \psi_n}{\partial x^2} + 0 = E_n \psi_n$$

Plug in $\psi_{n_{II}} = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi}{L} x\right)$

$$\longrightarrow \quad E_n = \frac{n^2 h^2}{8 m L^2} \qquad \boxed{n = 1, 2, 3, \ldots}$$

Or we could go back and just look at what we called k---back when it was first introduced.

- Why don't we need negative values of n----
  - They are redundant linearly dependent functions---n=-1 gives same as n=1 except for picking constant
- Why don't we need the n=0 value of "n"
  - We do sometimes----
  - Really we would go back to beginning and for region II we would ask "what is solution to diffeq for the case E=0"
  - It is not needed in this case---but sometimes it is needed (the E=0 solution)---in general don't forget it
    - Remember we just solved diffeq 101-------NOT 102,
- Since each full solution satisfies the diffeq -----can't I add solutions like a little bit of the n=1 function, plus a little bit of the n=3, and some n=4.....(any linear combination) and still solve the diffeq?

Continuing questions on solutions to SWE

- Can I take linear comb. of solutions
  - YES---Hell yes----
  - That is what a wave packet (particle) is all about--right?
  - WE MUST RECALL THAT THE PHYSICAL SOLUTION IS Ψ(x,t), ….not Ψ(x)--and that linear combinations are of the form

$$\Psi(x,t) = \sum_n A_n \psi_n(x) e^{-iE_n t/\hbar}$$

This leads to cross terms in time dependence. That is--time dependent probability distributions are composed of summing over STATIONARY STATE solutions (the eigenstates)--which each have different time dependence.

For example lets just add two such states below

We may take any superposition of "Eigenstates"

$$\Psi(x,t) = A_1 \psi_1(x) e^{-i\omega_1 t} + A_2 \psi_2 e^{-i\omega_2 t} + \cdots$$

Re-Normalize (divide by $\sqrt{A_1^2 + A_2^2 + \cdots}$

Note $\boxed{\Psi^* \Psi \text{ has time dep cross terms!}}$

The A's are there to renormalize ---we are reusing the letter A.

$$E_n = \frac{n^2 h^2}{8 m L^2}$$

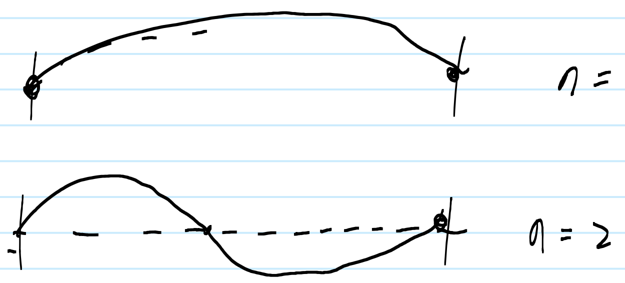
Note the result was put in terms of h, not h-bar.

For the case of the infinite square well---the energies, frequencies, and wavelengths relate and have pictures like waves on a string. ---same states---

Energies are discrete

$E_2 = 4E_1 \qquad E_3 = 9E_1 \qquad$ etc.

Standing wave solutions



Note

L is filled with $n\lambda_n/2$

or

$$\lambda_n = \frac{2L}{n}$$

recall $\quad k_n = \frac{\pi n}{L} = \frac{2\pi}{\lambda_n}$

wavenumber

Just as with waves on a string---the infinite potentials at the walls---hold the wavefunction down tight at the ends (x=0 and L). There are an integer number of half wave cycles between 0 and L for stationary state solutions.

IF I CHANGE THE LOCATION OF THE METER STICKS---SO THAT LEFT EDGE IS NOT AT ZERO---THE PICTURES FOR THE SOLUTIONS CANNOT CHANGE (HOW WOULD THE WELL KNOW I MOVED METER STICKS? SOLUTIONS--LOOK DIFFERENT FORMULA

We will now extend solutions to other picture wells, barriers, non-infinite.

We will examine:

- Setting up the equations (B.C.) leading to solutions
- And examine solutions
- We will often skip over math intermediates (quantum course)
- We need to focus on the meaning of some answers.

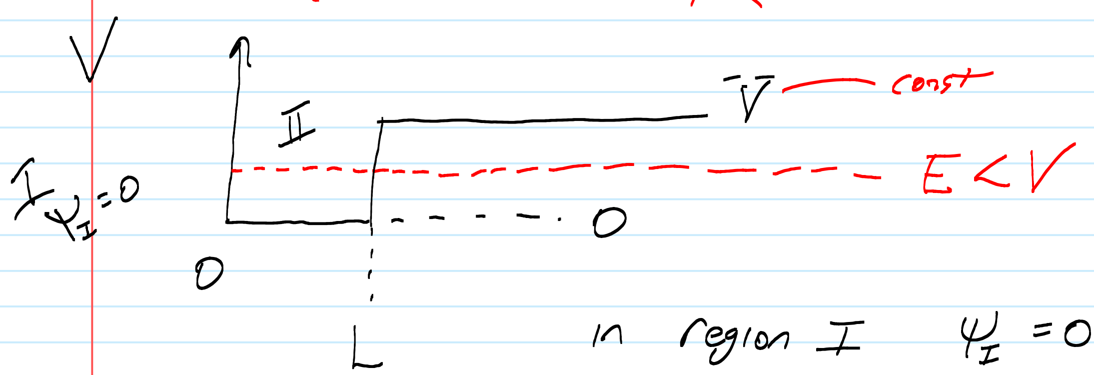
SEMI-Infinite well

done with simplest case — Now alter the well

The ∞ well had hard walls $F_x \propto -\partial V/\partial x$

Lets get rid of that (at Least somewhere)

infinite force



in region I $\psi_I = 0$

region II $\psi_{II_n} = A_n \cos(k_n x)$

OR $B_n \sin(k_n x)$

Note in region I that V(x)=infinite---so wavefunction is zero there. In region II V(x)=0 ....so far I don't need a symbol for the potential. In region III V(x)=V (probably should be V_o)
--it is the only "V" around today, and is constant. --Special case.

And from S.E. region II $\underline{\underline{k_n}} = \sqrt{2mE_n}/\hbar$

For region with $V = 0$

region II —

For region with $\overline{V} = 0$

In region III where the constant potential is V, then the SWE looks like.

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (E-V)\psi$$

or

$$\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} = (V-E)\psi$$

const & V-E = +

$$\kappa_n = \sqrt{\frac{2m(V-E)}{\hbar^2}}$$

In region III $\psi_{III_n} = C_n e^{\kappa_n x}$ or $D_n e^{-\kappa_n x}$ do it

$\kappa_n = \sqrt{2m(V-E)}/\hbar$

(from $\frac{\hbar^2}{2m}\frac{d^2\psi_n}{dx^2} = (V-E_n)\psi_n$)

(could use sinh or cosh)

Summary--

- In region I the form of wavefunction=0
- In region II---same FORM as before
- In region III we have introduced kappa to make constant real---and explicitly yield correct growth/decay functional form in this region.
- NOW apply BC Boundary I to II, then Boundary II-III (and implied boundary III to infinity---never blow up at inf).

B.C. ① $\psi_I(0) = \psi_{II}(0)$

also the constant term we neglected ... for n=0

$$0 = A_n\cos(k_n 0) + B_n\sin(k_n 0)$$

$$0 = A_n \, 1 \longrightarrow \boxed{A_n = 0}$$

We applied a boundary condition at x=0--and found constant An=0. We don't know ks or Bs.

We will now apply the boundary condition at x=+infinity.

- But Dr. Colbert---why, why are you taking us to infinity now? Why? https://www.youtube.com/watch?v=Tjqq3ElVYlg
- I thought up a reason I thought it up quick--
- There is a zero on one side---We'll fix it up there and bring it back here.
- Really---I can see a zero come up by applying the B.C. at infinity--Zero's knock out terms, reduce clutter, make the math cleaner.

B.C. as $x \to \infty$

ψ must be finite $\rightarrow$ $\boxed{C_n = 0}$

Go back and look at solution we wrote for region III
Set =not infinite at x=infinity. The term that wouldst blow up ---must go.
We can't have infinite probability of finding particle at infinity. That would not be normalizable.

B.C. at $x = L_-$ & L_+

$$\psi_{n_{II}}(x=L_-) = \psi_{n_{III}}(x=L_+)$$

We have the An's=0 and the Cn's=0----so the solution at x=L on either side so far looks like. And then derivative terms.

$$B_n \sin(k_n L) = D_n e^{-\varkappa_n L}$$

B.C. $\psi'_{II} = \psi'_{III}$ at L

$$k_n B_n \cos k_n L = -\varkappa_n D_n e^{-\varkappa_n L}$$

for $E_n < V$ divide eqs to get

$k \neq \varkappa$

(each has $|E-V|$ in a given region

$$\frac{\tan(K_n L)}{K_n} = \frac{1}{-\kappa_n}$$

Trans... - Plug in the given V, consts find E_n numerical

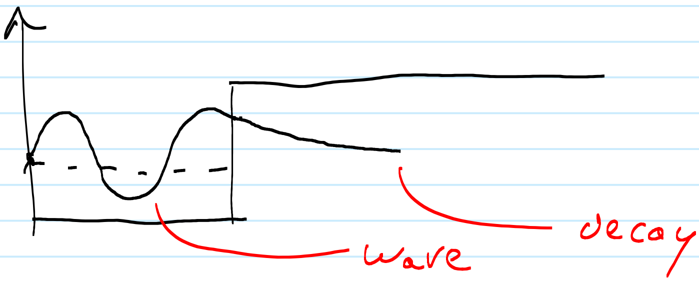
must redo for E > V (sometime)

Yes--this is a transcendental function ---it can only be solved numerically. So we are "given" a particular potential (V) and then find the allowed BOUND ENERGIES. We get different functions for the case E>V (then no kappa---)

once we find E_n (finite # of bound states!)

we can find ψ_n

and then normalize (not easy)



Notables:

- The numerical solutions will find limited number (not infinite) of Energies that work---above some max---NOT BOUND
- The pictures still look like "half wave-ish", "two half wave-ish", "three half wave-ish" but stretched longer
    - Longer waves mean lower energies than the infinite well
    - Part of the wavefunction is outside the well
    - There is a probability of finding the particle inside the barrier
        - Where the Kinetic energy is negative---Yeah--I said it.

The energies are lower than for equiv n of ∞ well (since λ leaks out, is longer, E_n is less)

$\psi \neq 0$ for region to right of well.

Larger L, longer λ, $\rightarrow$ Lower energy

There are a finite # of bound states

— there is an n_{max}

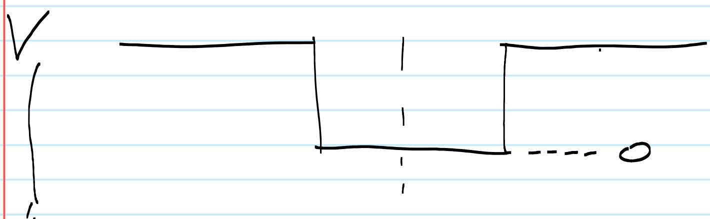
Wait---can't I make the energy anything I want --?

No---the wave interferes itself away unless it meets standing wave like conditions (B.C.)---sorry

The finite well---https://www.youtube.com/watch?v=ud1zpHW3ito

14.3 Finite Well —

there are other square well systems



Can change height of either side ...

Bad notation but —

$$V_I(x) = V \leftarrow \text{constant} \qquad V_{III}(x) = V$$

$$V_{II}(x) = 0$$

So this is a finite well, the sides (edges) might be at say 0 to L or -L/2 to +L/2---but me moving the meter stick location does not change the pictures. We will consider solutions for bound state energies only E<V. When E> V--the particle will be free.

- We know the form of solutions in each region I, II, III---
- We know to impose the B.C. at infinity---either + or - and to start out by omitting the offending "blows up" term.

We know solutions to diffeq

I) $\psi_{I_n} = C_n e^{+\varkappa_n x}$

II) $\psi_{II_n} = A_n \cos k_n x + B_n \sin k_n x$

III) $\psi_{III_n} = D_n e^{-\varkappa_n x}$

$$\underbrace{\varkappa_n = \sqrt{2m(V-E)}/\hbar \qquad k_n = \sqrt{2m(E_n - 0)}/\hbar}_{\text{from S.E.}}$$

we dont know energies yet ....

So, just starting here--we have A's, B's, C's, D's, and Energies(n, k's)---to find. Five (sets) of constants.

How can we find five constants?
We must have five equations.
ψ and ψ' continuity at both x= left, right edges.
AND---NORM.

finding C_n, A_n, B_n, D_n, E_n shall come from applying the remaining B.C.

(which did we do already ?)

Answers will depend on V, L
— takes some algebra

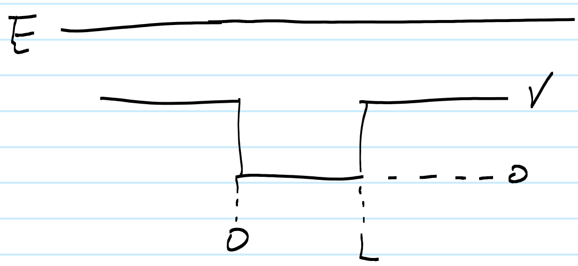
ψ at 0
ψ' at 0
ψ at L
ψ' at L
& NORM

Hard — Wait for Quantum.
There will be a finite # Bound States!

We can set up the equations----and then give some results or discuss. ---meaning, be prepared to do that.

Now consider $E > V$

Free Particle--not bound



This is somewhat analogous to light going through a plate of glass — or converse — light through a plate of vacuum!

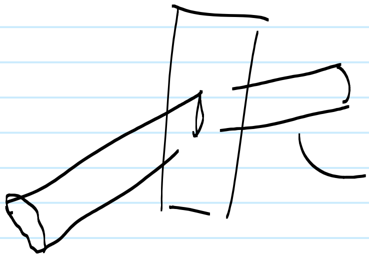
We select a wave incident from left to right
We select λ and amplitude
A^2 = intensity

For the Free particle case ---(really always) we are free to make some choices in the offset of potential.
We could also have a barrier rather than a well.

This free particle case--the particle behaves like "optics"

What happens at each boundary
- some reflection
- some transmission

Note



V is part of intensity as "sausage of light" may expand or contract

Upon entering the new medium
λ changes and also speed changes.

back to

I shall decide to send a wave in from left to right perhaps

$$\psi_I = A e^{ik_I x} + B e^{-ik_I x}$$

wave sol everywhere
- no "n"s though we can

$$k_I = \sqrt{\frac{2m(E-V)}{\hbar^2}}$$

recall $\Psi = \psi e^{-i\omega t}$ → right and left going waves
$e^{i(kx-\omega t)}$, $e^{-i(kx+\omega t)}$

A is what I select going in —
B is determined by reflection

So, I can decide what to make A ---the left going wave is determined by reflectionsssssss. A somehow indicates the intensity or flux of particles entering. We expect there will be some wave reflection at the first interface (and transmission), and some additional wave reflection at the second interface (and transmission). The reflected waves may interfere. So ---again at either interface there is a ---right going wave and a left going wave----

https://www.youtube.com/watch?v=dZmZzGxGpSs

$$\psi_{II} = C e^{iK_{II}x} + D e^{-iK_{II}x}$$

(we decided to write waves like this ! Easier)

$$K_{II} = \sqrt{\frac{2mE}{\hbar^2}}$$

and in region III

$$\psi_{III} = F e^{iK_I x}$$

(note $K_I = K_{III}$)

right going — no reflection off $x \to \infty$

Five constants B, C, D, F — A we decide

4 B.C. eqs + Normalization (done by this)

The k's in each region were known. Any energy can be selected for free particle systems.

Will really solve for things like

B/A, C/A, etc 4 ratios to find

at $x=0$ $\psi_I = \psi_{II}$ $\psi_I' = \psi_{II}'$

$\rightarrow$

$$\boxed{\begin{aligned} A + B &= C + D \qquad (e^{i0} \rightarrow 1) \\ \text{and} \quad k_I A - k_I B &= k_{II} C - k_{II} D \end{aligned}}$$

and now at $x = L$ $\psi_{II}(L) = \psi_{III}(L)$ $\psi_{II}'(L) = \psi_{III}'(L)$

$$\boxed{\begin{aligned} & C e^{ik_{II}L} + D e^{-ik_{II}L} = F e^{ik_I L} \\ & \text{and} \\ & k_{II} C e^{ik_{II}L} - k_{II} D e^{-ik_{II}L} = k_I F e^{ik_I L} \end{aligned}}$$

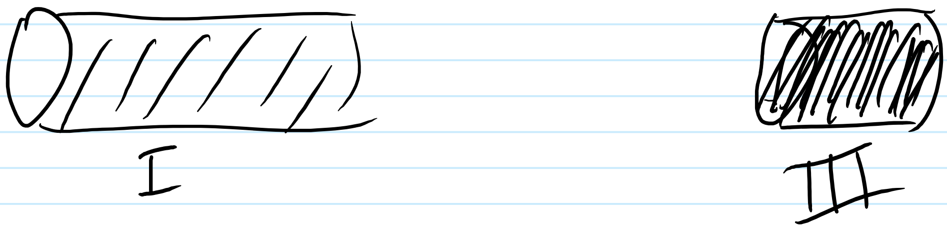
4 eqs 4 unknowns —
know how to write out & where from.

solve for

Transmission past both surfaces and also Refl. here - speed doesnt matter since region I has same potential as III

$$T = \frac{\psi_{III}^* \psi_{III}}{\underset{\substack{\text{right}\\\text{going}}}{\psi_I^*} \underset{\substack{\text{right}\\\text{going}}}{\psi_I}} = \left(\frac{F}{A}\right)^* \left(\frac{F}{A}\right)$$

Note---that since k(I) and k(III) are the same----then the bundle of energy that got through is the same length. If the potential in region III differs---then the speed changes. If the particle beam is travelling slower in region III (and the beam is continuous), then the relative intensity will be greater.



In more general cases you will see a relative speed come into the Transmission term.

$$R = \frac{\psi^*_{I_{Left}} \, \psi_{I_{Left}}}{\psi^*_{I_{Right}} \, \psi_{I_{Right}}}$$

Inside the well there are interference terms & would describe energy density moving either way

$$T = \left[1 + \frac{(V \sin k_{II} L)^2}{4E(E-V)} \right]^{-1}$$

I've given the answer for this case. k_{II} is known (given/chosen). And we should note that T +R =1 (rate at which energy enters =rate at which it leaves--once in steady state).

So once we know Transmission---we know Reflection (and vice versa).

THERE ARE VALUES OF k_{II} L for which T= either 0 or 1.
Look at the sin term.
I can construct systems with either full or zero transmission!!!!!!

Note — when $\sin K_{II} L = 0$

that $T = 1$ full transmission

for $K_{II} L = n\pi$

$$\frac{2\pi L}{\lambda_{II}} = n\pi$$

or $2L = n\lambda$ $\quad n = 1, 2, 3 \ldots$

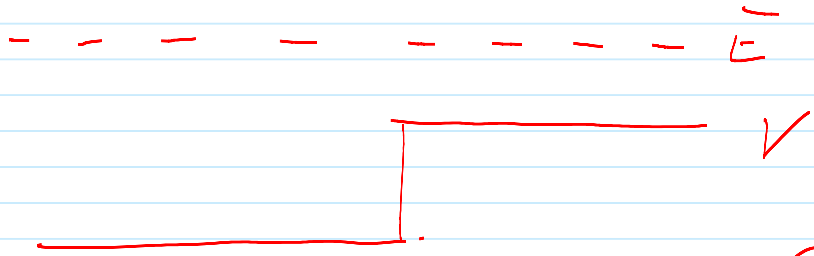
Full transmission when $L = n\left(\frac{\lambda}{2}\right)$

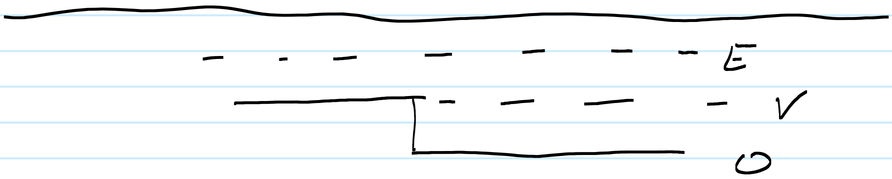
Note — Very similar to optics!
Phase changes may occur at one surface or another

So, the wavelength and energy are related by k in the material. We pick what energy (frequency) to send in---and this (and material properties ---which is "V" here) determines wavelength in all regions. Then by picking "L"---properly we can get full transmission. ANTI REFLECTION COATED OPTICS----
https://static1.squarespace.com/static/592e542a579fb3611cbeaf8c/t/594769be1e5b6c4c4dd1746f/1497852354059/Ngomad+Mobile+Display+Enhancer+-+Technical+Bulletin.pdf

We could go back and redo

or





etc

We could use the same techniques to analyze these systems and more. We can add little defects in a region (perturbation) and get approximate impact of those (little bumps in potential).

.....and more.