Ch. 12

Bohr's model for hydrogen was an add on to classical physics.
It is not perfect--but there are some "clean ups " we can do.

I WILL FIX UP THE BOOK CONFUSION ON RESULTS.

THE PREVIOUS CHAPTER 11 RESULTS FOR INFINITE MASS NUCLEUS WERE.

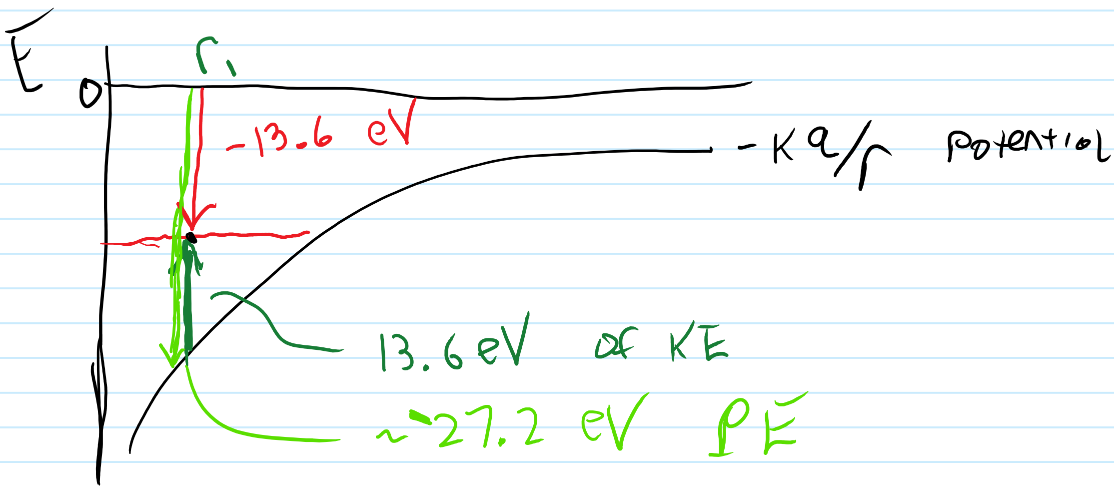
$$E_{n\ \infty \text{ mass nucleus}} = -\frac{Z^2}{n^2}\,13.61\ \text{eV}$$

THIS IS HOW EQUATION 11.4 SHOULD READ---AFTER FAILING TO ACCOUNT FOR REAL MASS OF NUCLEUS, AND ROUNDING PROPERLY IN THAT LAST DIGIT. AS WE ACCOUNT FOR REAL NUCLEAR MASS IN HYDROGEN WE WILL GET 13.60 (IT IS ABOUT A 1/1800 FACTOR DIFFERENCE DUE TO MASS OF PROTON).

ALL OTHER ATOMS HAVE SMALLER EFFECT SINCE THEIR MASSES ARE CLOSER TO INFINITE.

ENERGY LEVELS

The energy levels suggested by Bohr for Hydrogen like atoms---are called stationary states. These are states where the electrons keep their energies.



In that last figure I have shown the ground state energy inside the Coulomb potential for hydrogen. The KE is half the size of the PE in the well, the orbit is circular and Bohr has suggested that light is emitted (or absorbed) as electrons make an ELECTRONIC TRANSITION FROM ONE ENERGY LEVEL TO ANOTHER.

$$|E_{n_f} - E_{n_i}| = h\nu$$

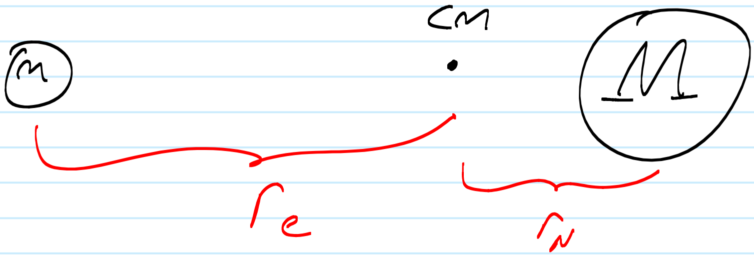
The correction for "reduced mass" of the nucleus (real non-infinite mass), in the case of hydrogen has about a 1/1800 part correction. This is pretty easy to see with common optical spectroscopy equipment.

REDUCED MASS Correction

election mass $= m$

Nucleus mass $= M$

The nucleus could have a single proton, or ....on up.



$$r = r_e + r_N$$

$$r_N = \frac{m r}{M + m} \qquad r_e = \frac{M r}{m + M}$$

do it
- define
CM

So in our model, both objects rotate around the common center of mass point. The radius of the orbits are r_e r_N and r respectively (see figure above).

Now we will go back to Bohr's model and insert our new "orbit" information that includes the real masses. Previously---the fact that we said the nucleus did not move is equivalent to saying it has infinite mass.

is mv^2/r but r is out front –

$$\underbrace{\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}}_{\text{force on } e^- \text{ from object at distance } r} = \underbrace{m r_e \omega^2}_{e^- \text{ moves in circle of this radius}}$$

We can see that the distance between positive and negative (.r.) is not the same as the electrons orbital radius. Fixing the above expression for r_e we get

$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = m\left(\frac{M}{m+M}\right) r\omega^2$$

And we see already things are different. We need to impose Bohr's quantization rule now---but we have two masses contributing to the angular momentum, so.....adding. Recall that ---for a point object, the angular momentum can be written as "mvr". (and $v=r\omega$)

$$(m v_e r_e + M v_N r_N)\, 2\pi = n h$$

or

$$\underbrace{(m r_e^2 \omega + M r_N^2 \omega)\, 2\pi}_{\text{reduces to}} = n h$$

using CM expressions for r_e, r_N

$$\frac{mM}{m+M} r^2 \omega \, 2\pi = nh$$

$\frac{mM}{m+M}$ → called Reduced Mass m_r

So, if we simplify our expressions using the reduced mass definition m_r then we get two fairly simple starting equations to work with, and we go about finding radii, speeds, energies ---much as we did previously.

$$(m_r r^2 \omega) 2\pi = nh$$

and
$$\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} = m_r r \omega^2$$

eliminate ω to get r

$$r = \frac{n^2 \epsilon_0 h^2}{Z m_r \pi e^2}$$

New result for r

If we now use the reduced mass expression and relation between r_e and r, we get for the radius of the electrons orbit

$$r_e = \frac{n^2 \epsilon_0 h}{Z m \pi e^2}$$

THIS IS THE SAME RESULT WE HAD BUT IS NOT THE WHOLE PICTURE OTHER MASS IS MOVING TOO.

We will continue and get the energies out of this---you can get ω now, and $v=r\omega$ if needed. But we head to ENERGY.

NOTE--we are simply providing for a correction. Next semester you will do a much lengthier and more appropriate solution to Schrodinger's wave equation to solve for all the properties of Hydrogen--still not perfect.

r_e will only change if fundamental constants change.

Now Energy:

$$KE = \frac{1}{2} m r_e^2 \omega^2 + \frac{1}{2} M r_N^2 \omega^2$$

$$= \frac{1}{2} m_r r^2 \omega^2$$

recall $\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r^2} = m_r r^2 \omega^2$

$$= \frac{1}{8\pi\epsilon_0} \frac{ze^2}{r}$$

$$PE = -\frac{1}{4\pi\epsilon_0} \frac{ze^2}{r}$$

$\left(|KE| = \frac{1}{2} PE \right.$ circular orbits $\left. \right)$

This all looks pretty close to what we had before--it is except for perhaps the mass and r differ. Putting it all together (KE and PE added, then plugging in r) we get

So we still have

$$E_n = -z^2 \left(\frac{m_r e^4}{8 \epsilon_0^2 h^2} \right) \frac{1}{n^2}$$

Similar but now with m_r not m_e

So the big difference is going to be due to the reduced mass. Note that z^2 and n^2 are in here in the same way. The only change is "m_r" ----so we can crunch this out for "real hydrogen"

Real hydrogen has the ---LEAST INFINITE MASS LIKE NUCLEUS OF ANY ATOM---So Real Hydrogen will have a spectra that has more closely spaced energy levels than we previously had.

Consequences of REAL hydrogen compared to fictional "infinite mass nucleus" hydrogen.
Real hydrogen has

- Smaller size energies, and smaller separations of energy levels
- The frequencies will be smaller for Real hydrogen
- The wavelengths will thus be longer for real hydrogen

You should think about where real hydrogen spectral lines and energy levels will be for comparing Regular old hydrogen (one proton), to Deuterium, or Tritium ---think about it.

The spectral scan below shows two of the Balmer Series lines for Hydrogen and Deuterium.