Ch. 10 Wave-Particle

We have seen that electromagnetic waves come with in quantized packages--photons---that carry particle properties. The photons have discrete lumps of --energy, momentum, and angular momentum.

Since our waves have particle like properties---
It is now time to ask the question--and recognize experimental results.

DO PARTICLES HAVE WAVE PROPERTIES?

For photons we had for momentum

$$p = \frac{h}{\lambda} \quad \text{or} \quad \lambda = \frac{h}{p}$$

We got this result from $E=h\nu=hc/\lambda$ and $E^2=p^2c^2+0$---for photons.

For photons we know what is waving. We also saw that for particles travelling the speed of light---those had to have zero rest mass (or vice versa).

We are going to suggest now that all particles ---everything-- has some kind of wave property with a wavelength determined by

$$\lambda = \frac{h}{p} = \frac{h}{mv}$$

For photons we know what is waving (electromagnetic fields)--- now---I am not sure what the waving stuff is (I'll need to learn it's properties)--but the wave experiments do indeed measure waves.

The wavelength are denoted as DeBroglie wave/wavelength

The waves are observed with
- Atoms
- Electrons
- Any particle for which I can do experiments like optical interference (double slit, many slits, or reflective cavities)
- The waves are called matter waves---whatever that will come to mean.

COMPLEMENTARITY
- We may do experiments that fundamentally demonstrate EITHER
  - particle properties OR
  - wave properties BUT
  - We may not yield results of both properties at the same time
  - Both sets of properties exist for any "thing"---
    - Thing being mass energy stuff---
- Maybe life is like a box of chocolates---or maybe like a feather floating on the wind (maybe it's a little bit of both).
- Wave properties tend to be indeterminate---but are probabilistic
- Particle properties tend to have definite values (as well as can be measured --there is always a limit).

If you wish a philosophical discussion here regarding determinism/causality/clockworkism/Everything happens for a reason(scientifically) (particle nature) VS. probability, indeterminate, free will, choice, and other religious hokum----Pause----stop----give up-----and simply be empirical.

10.3 WAVE PACKETS

Some notes about waves before adding waves

A wave of some shape can be written as $f(x-vt)$ for right going wave in one dim, or $f(x+vt)$ for left going wave.

We can insert a constant in the argument (x-vt) without changing anything. If I multiply by constant---it is still a function of x-vt.

$$\frac{1}{\lambda}(x - vt) = \frac{x}{\lambda} - \frac{v}{\lambda}t$$

Of course, v/λ=frequency ν---but if I throw another constant in---say 2π then 2πν is simply the angular frequency ω

$$\left(\underbrace{\frac{2\pi x}{\lambda}}_{k} - \underbrace{\frac{2\pi}{T}}_{\omega} t\right) \qquad T = \frac{1}{\nu}$$

Wave number/vector		Angular frequency

There are other meaning
for wave number (a unit)….different.

So---big people---past intro physics typically write their waves as functions of kx-ωt (sometimes ωt-kx --no matter).

$$f(kx - \omega t)$$

While the shape of a wave packet may be anything---we often talk about a pure single frequency wave or at least simple waves added up.

$$\sin(kx - \omega t)$$

or $\cos(kx - \omega t)$

or $e^{i(kx - \omega t)}$ Say Euler---

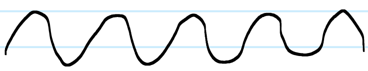
All of these are good ways to write a simple wave.

If we want to add waves to make a discrete style packet---like a particle---then we can do this mathematically---ADDING

GO TO
https://phet.colorado.edu/sims/cheerpj/fourier/latest/fourier.html?simulation=fourier

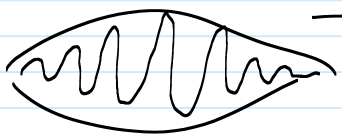
Now we are going to add some waves---to keep it to the bare minimum, lets add two pure sin waves. But they may have slightly different wavelengths--frequencies.

How do we describe waves & wave prop.

$-\infty \leftarrow$  $\rightarrow \infty$

must extend forever to have "pure" ν

Add frequencies to create wave packet



$\longrightarrow$ Packet moves

— Phase may change within

We will add two waves---one with wavelength λ or really

$$K = \frac{2\pi}{\lambda} \quad \& \quad \omega = \frac{2\pi}{T}$$

and $K + dK$ $\quad\quad \omega + d\omega$

for the second wave

Note that $\omega/k = v$ speed of some--non thing--somekind of v

$$\left.\begin{aligned} \psi_1 &= A \sin(Kx - \omega t) \\ \psi_2 &= A \sin\left((K+dK)x - (\omega + d\omega)t\right) \end{aligned}\right\}$$

We let A be same here

Interference — Add

$$\sin\theta + \sin\phi = 2\cos\left(\frac{\phi-\theta}{2}\right)\sin\left(\frac{\phi+\theta}{2}\right)$$

$$\psi = \psi_1 + \psi_2 = 2A\cos\left(\tfrac{1}{2}[dk\,x - d\omega\,t]\right) \times \sin(\bar{k}x - \bar{\omega}t)$$

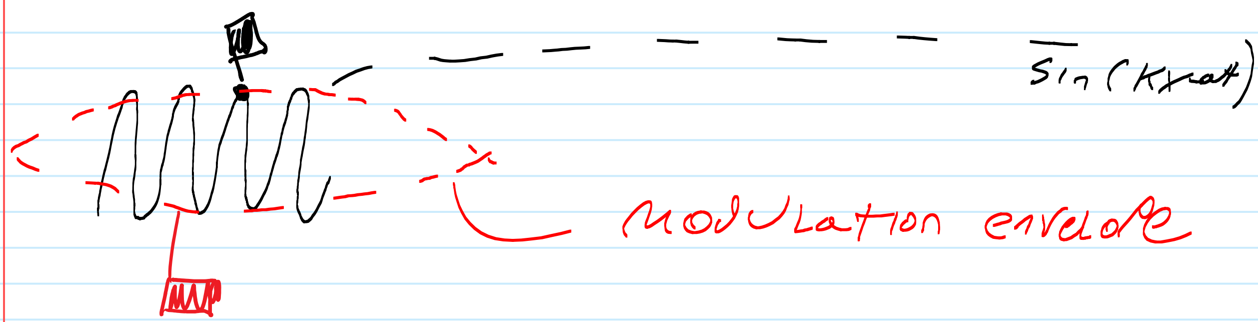
halfway between $\left\{ \begin{array}{l} \bar{k} = k + \frac{dk}{2} \\ \bar{\omega} = \omega + d\omega/2 \end{array} \right.$

In the sin term--we can drop the dk/2 and the dω since they are very small compared to the k and ω. So the sin term has big frequencies---spatially and temporally---it oscillates rapidly.

The cosine term has the much smaller dk and dω only--so oscillates very slowly.

The end result looks like

The cosine term amplitude modulates the sin term. And we have a wave packet. Demonstration that it is possible to construct a "particle" by combining waves with differing wavelengths, amplitudes, phases.



I've drawn a black flag on a peak---and will allow it to move with that peak.   and I'll ask --how fast it moves

The red flag is tied to the outer modulating envelope--and I'll want to know how fast that is also.

They are different.

To determine the speed of the BLACK FLAG..we look at the sin term   sin(kx-ωt)

The term in parentheses is called "phase" ---as a name for the argument of the sin function.   It is an angle.

To keep our flag on the peak as the peak moves---then kx-ωt   must remain constant.   As x advances (making kx bigger) then ωt increases too (it takes time for the wave to move)....and the phase remains constant (to describe the same peak on the wave----something like   π/2   is the name of that particular black flag.

$$\frac{d}{dt}\left[\text{Const Phase} = Kx - \omega t\right]$$

$$\underbrace{\frac{d(\text{Phase})}{dt}}_{0} = K\frac{dx}{dt} - \omega$$

$$V_{ph} = \frac{\omega}{K} = \underbrace{\frac{\cancel{2\pi}}{T}\,\frac{\lambda}{\cancel{2\pi}}}_{V}$$

This is the normal phase velocity--what we have always thought of as "wave velocity".

but we also want to know how the outer cosine envelope moves (the packet/the particle/the group)

For group velocity---same argument, but cosine term.

The "phase" of the cosine term is given by

$$\frac{d}{dt}\left[\underset{\uparrow\ \text{const}}{\text{group Phase}} = \frac{1}{2}dK\,x - \frac{1}{2}d\omega\,t\right]$$

$$0 = dK\,V_g - d\omega$$

$$V_g = \frac{d\omega}{dK}$$

It will turn out (we'll show)---that v_{phase} can be anything ---any speed---infinite is OK too. Instant --across the universe.

But $v_g <= c$ Energy, stuff, information, etc---can't move faster than light.

Consider some – ? wavicle ?

$$\omega = 2\pi\nu$$

$$= 2\pi\, E/h$$

$$\boxed{d\omega = \frac{2\pi}{h}\,dE}$$

$$K = \frac{2\pi}{\lambda} = \frac{2\pi}{(h/p)}$$

$$= 2\pi\, p/h$$

$$dK = \frac{2\pi\, dp}{h}$$

So from results on last page---the group velocity is

$$V_g = \frac{d\omega}{dK} = \frac{dE}{dP}$$

Now consider just a relativistic moving particle

$$\frac{d}{dP}\left(E^2 = P^2c^2 + E_0^2\right)$$

$$\cancel{2}E\left(\frac{dE}{dP}\right) = \cancel{2}P c^2$$

$$\frac{dE}{dP} = \frac{P\cancel{c^2}}{m\cancel{c^2}} = V$$

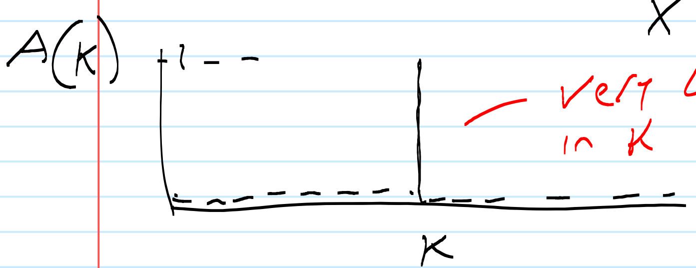
This is the speed of the particle in relativity---and always must be less than (or equal to) c.

10.4 Fourier----adding waves---Physics is about SUPERPOSITION (adding of waves).

Heisenberg ——

Here is a single pure wave--amplitude vs position at a given time

No spatial confine



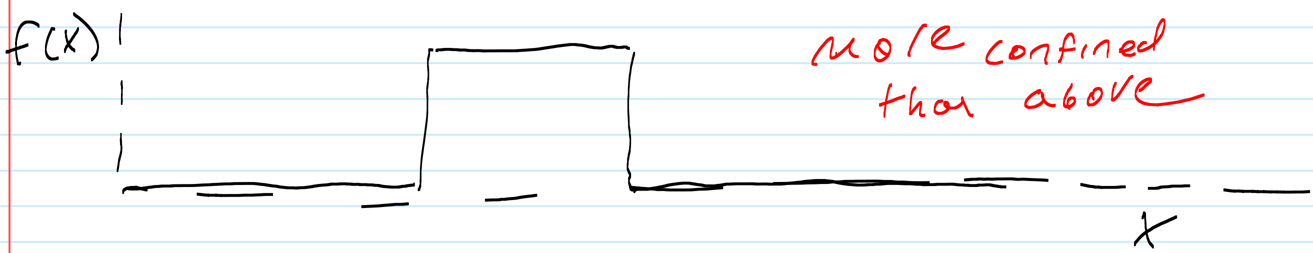
— very confined in K (or ω)

Fourier spectrum of sin(KX)

The pure wave--which is spread out over all of "real space" x---is confined when plotted in frequency (or wavenumber k) space. If I have more than one frequency the wave will be confined to a packet --spatially--as we saw.

If we know the frequency well---we don't know the "where" at all. If we know where---then conversely, we do not know the frequency.

We can examine many systems this way (and you have)-will.



What does fourier spectrum look like

In electronics you may talk about Fourier synthesis--adding discrete waves to get a repeated wave form (sawtooth, triangle, whatever shape). You may do this in math methods. You may do this in mechanics (wave on string).

We can add waves with standing wave modes like----

$$f(x) = \sum_n a_n \cos(k_n x) + b_n \sin(k_n x)$$

$$k = n\frac{\pi}{L} \quad \text{for some } L$$

or

$$= \sum_n A_n e^{i(k_n x + \phi_n)}$$

Above are discrete sums of spatial frequencies (wavelengths)---or we might have a continuous sum (integral)

$$f_x = \int \underline{A(k)}\, e^{i(kx + \phi_k)}\, \underline{dk}$$

$$A(k)\,dk = \overset{\text{old}}{A_n}$$

Either way----if we add waves, we can confine the packet locally, and have sharp edges---but we need a broader range of frequencies to do this.

This can be done either spatially or in time.

Heisenberg (Say my name---https://www.youtube.com/watch?v=fHKrCs1rFRI---OK--not "that Heisenberg"---W.W.)
Uncertainty Principles

① $\Delta E\, \Delta t \geq h$ we can argue $\hbar/2$ etc

② $\Delta p_x\, \Delta x \geq h$

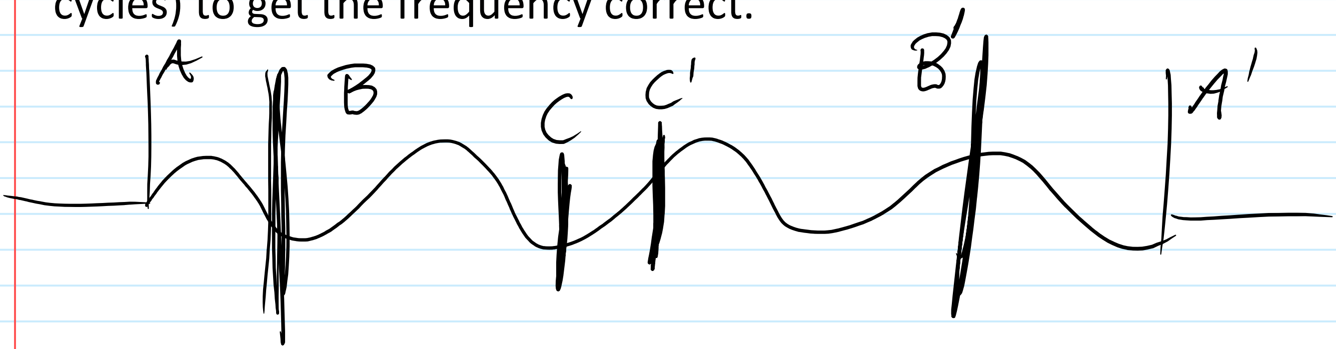
1

$$\Delta E \, \Delta t \geq h$$

$$h \, \Delta \nu$$

$$\cancel{h}\left[\Delta\left(\frac{1}{T}\right)\right]\Delta t \geq \cancel{h}$$

If we look at some wave packet made of many different frequencies---we need to measure it for a while (over many cycles) to get the frequency correct.



If I measure from A to A'---I have a pretty good idea of all the frequencies making up the packet (I know ν or energy really well---but it took some time). If I measure B to B' (shorter packet--takes more frequencies to make this packet---less defined frequency--broader frequency range--but less time). If I measure C C' I have no real clue as to frequencies---there is a big spread--many frequencies required to make up a short packet---big spread in ν when the time is short.

The factor of Planck's constant comes up as an empirically measured --observed quantization relating energy to frequency. But really what we are saying is that

$$\Delta \nu \, \Delta t \gtrsim 1$$

$$\Delta\left(\frac{1}{T}\right) \Delta t \gtrsim 1$$

Simply---I need many cycles to nail down the frequency well. But that takes time.

2

$$\Delta p \, \Delta x \geq h$$

$$\cancel{h}\left(\Delta \frac{1}{\lambda}\right) \Delta x \geq \cancel{h}$$

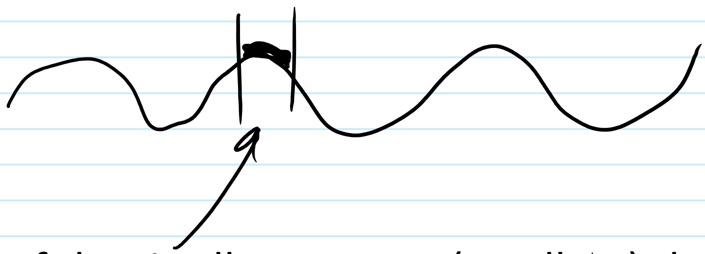
We get exactly the same discussion whether we are talking about a spatial cycle of waves a time cycle of waves.

Again--empirically recognized that momentum is quantized ---but let's stick to wavelength spread (in the energy time uncertainty that would be like a spread in the time period for different waves).

If I confine the wave in the x direction--then I cannot know the differing waves (moving) in the x direction.   I cannot measure exact position and also get the wavelength

Again the variables have inverse units

$$\Delta\left(\frac{1}{\lambda}\right) \Delta x \geq 1$$

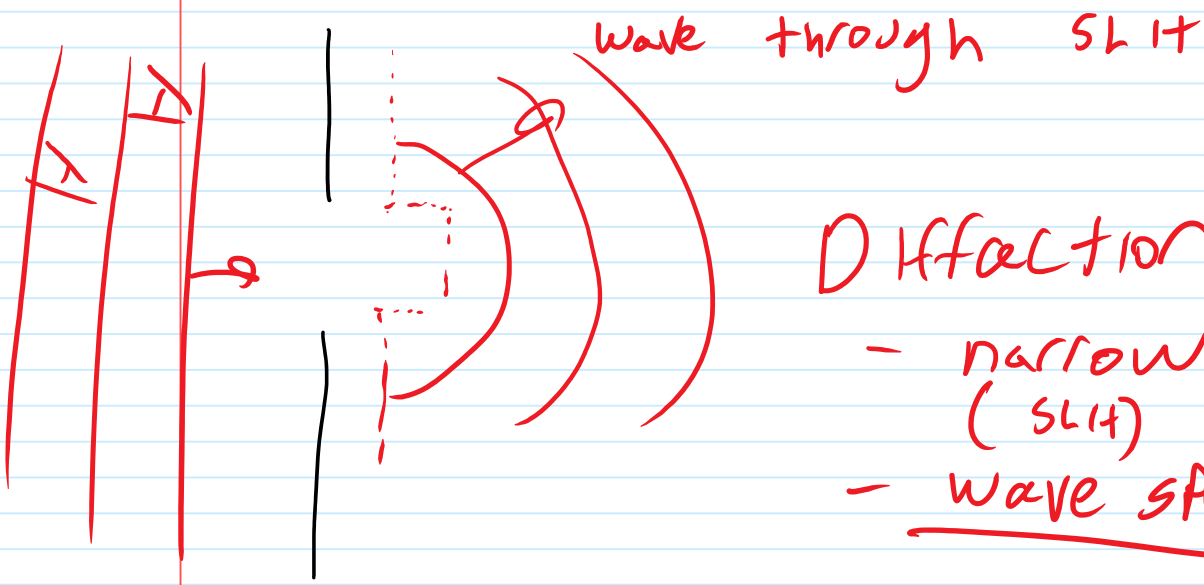


If that is all I measure (small Δx) then I don't know the wavelength very well---just looking at the picture, this seems intuitive. I must measure many cycles (big ΔX), in order to know the wavelength well (small spread in $\Delta 1/\lambda$)

Brief discussion of uncertainty principle jargon.

- All physical quantities that can be measured seem to have some quantization related--and all quantization statements seem to relate to Plancks constant
- As we attempt to measure many properties of a system we can pair quantities up.
    - The paired quantities are either "commuting" (we can measure both as precisely as we want with no physical limits
    - The paired quantities are "non-commuting" --in which case there is an uncertainty relation (involving Planck's constant).
- Planck's constant h---has units of angular momentum   Js

Uncertainty in non-commuting variables is another way of simply saying ---if we confine the wave, we can't know the wavelength very well (https://www.youtube.com/watch?v=Kz40vwcTGFo)



Diffaction
- narrow Δx (slit)
- wave spreads