

Section 6.3

Double-Angle, Half-Angle, and Power-Reducing Formulas

Double-Angle

$$\begin{aligned}\sin 2\theta &= 2 \sin \theta \cos \theta & \cos 2\theta &= \cos^2 \theta - \sin^2 \theta \\ & & &= 1 - 2 \sin^2 \theta \\ & & &= 2 \cos^2 \theta - 1\end{aligned}$$

These identities are derived from the addition identities and the fact that $2\theta = \theta + \theta$.

Half-Angle

$$\sin \frac{u}{2} = \pm \sqrt{\frac{1 - \cos u}{2}} \qquad \cos \frac{u}{2} = \pm \sqrt{\frac{1 + \cos u}{2}}$$

These identities are derived from the $\cos 2\theta$ identities and the substitution $u = 2\theta, \frac{u}{2} = \theta$.

Traditionally, these identities were used to calculate sines and cosines of smaller and smaller angles.

The sign in front of the radical depends on the quadrant that the terminal side of $\frac{u}{2}$ lies in.

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Power Reducing

$$\sin^2 \theta = \frac{1}{2}(1 - \cos 2\theta) \qquad \cos^2 \theta = \frac{1}{2}(1 + \cos 2\theta)$$

These identities are rearrangements of the $\cos 2\theta$ identities.

As the name implies, they are used to reduce the powers on trigonometric expressions. This is a helpful simplification in calculus.