Distributing and Parallelizing Non-canonical Loops

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Loop Optimization

```
loop (0...n) {
    task_x
}
loop (0...n) {
    task_y
}
```

Fission or distribution

⇔

```
loop (0...n) {
    task_x
    task_y
}
```

Fusion or combination

⇔

```
loop (0...n/2) {
    task_x
    task_y
}
loop (n/2...n) {
    task_x
    task_y
}
```

Splitting

. . . and many more strategies.

In This Work

We present a loop optimization algorithm based on **loop fission** transformation, to introduce **parallelization potential** in previously uncovered cases.

Potential for parallelism

- Identify *independent* operations
- Perform those operations in any order as system resources become available

Loop fission (or distribution)

- Break loop into multiple loops
- Each loop has the same iteration range
- Each takes part of original loop's body
- Some duplication may be needed

Conceptually: Distribute loops $\Rightarrow$ parallelize $\Rightarrow$ speedup in execution time

Our Technique

- Is applicable even when iteration space is unknown.
- Can be applied to any kind of loop: `for`, `while`, ...
- Can be applied to languages from high-level to intermediate representation.
- Is suitable for integration with automatic compilation and optimization tools.

Technique Overview

Start with a sequential imperative program.

1. Perform dependency analysis using data flow graphs (DFGs).
2. Build a dependency graph.
3. Compute condensation graph and compute its covering.
4. Create loop for each statement in covering.
5. Parallelize distributed loops.

Program Under Analysis

We consider simple deterministic imperative `while` language, with variables, expressions, commands, and parallel command. Program can include:

- Arrays and pure function calls,
- Arbitrarily complex update/termination conditions,
- Loop carried-dependencies, and
- Arbitrarily deep loop nests.

Certain memory accesses are out of scope: pointers, aliasing, etc.

Variables in Command $\texttt{C}$

We identify variables modified by (Out), used by (In), and occurring (Occ) in $\texttt{C}$.

E.g., $\texttt{C} ::= \texttt{t[e}_1\texttt{]} = \texttt{e}_2$,

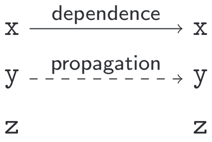
$$\begin{aligned}\mathrm{Out}(\texttt{C}) &= \texttt{t}\\ \mathrm{In}(\texttt{C}) &= \mathrm{Occ}(\texttt{e}_1) \cup \mathrm{Occ}(\texttt{e}_2)\\ \mathrm{Occ}(\texttt{C}) &= \texttt{t} \cup \mathrm{Occ}(\texttt{e}_1) \cup \mathrm{Occ}(\texttt{e}_2)\end{aligned}$$

We represent and analyze these dependencies using Data Flow Graphs (DFGs).

Data Flow Graph (DFG)

- A DFG is a matrix over a fixed semi-ring.
- Represents a weighted relation on set of variables involved in command `C`.
- 3 types of dependencies:

∞	dependence
1	propagation
0	reinitialization



Constructing DFGs

For each command, we define a mapping from variables of command $\mathtt{C}$ to DFG.
We write $\mathbb{M}(\mathtt{C})$ for the DFG of $\mathtt{C}$.

Definition: Assignment

Given an assignment $\mathtt{C}$, its DFG is given by:

$$\mathbb{M}(\mathtt{C})(\mathtt{y}, \mathtt{x}) = \begin{cases} \infty & \text{if } \mathtt{x} \in \mathrm{Out}(\mathtt{C}) \text{ and } \mathtt{y} \in \mathrm{In}(\mathtt{C}) & \text{(Dependence)} \\ 1 & \text{if } \mathtt{x} = \mathtt{y} \text{ and } \mathtt{x} \notin \mathrm{Out}(\mathtt{C}) & \text{(Propagation)} \\ 0 & \text{otherwise} & \text{(Reinitialization)} \end{cases}$$

Representing DFGs

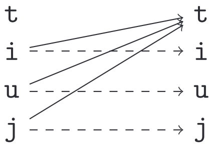
$$\texttt{C} ::= \texttt{t[i]} = \texttt{u} + \texttt{j}$$

$$\begin{aligned} \mathrm{Out}(\texttt{C}) &= \{\texttt{t}\} \\ \mathrm{In}(\texttt{C}) &= \{\texttt{i}, \texttt{u}, \texttt{j}\} \\ \mathrm{Occ}(\texttt{C}) &= \{\texttt{t}, \texttt{i}, \texttt{u}, \texttt{j}\} \end{aligned}$$

$\mathbb{M}(\texttt{C})$

$$\begin{array}{c} \\ \texttt{t} \\ \texttt{i} \\ \texttt{u} \\ \texttt{j} \end{array} \begin{array}{c} \begin{array}{cccc} \texttt{t} & \texttt{i} & \texttt{u} & \texttt{j} \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ \infty & 1 & 0 & 0 \\ \infty & 0 & 1 & 0 \\ \infty & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\mathbb{M}(\texttt{C})$ as a graph



Correction

All body variables of conditional and loop statements depend on its control expression. We apply loop correction to account for this dependency.

For e an expression and C a command, $\mathrm{Corr}(\mathtt{e})_{\mathtt{C}}$, is $E^t \times O$.

- E^t – column vector with ∞ for variables in $\mathrm{Occ}(\mathtt{e})$ and 0 for other variables.
- O – row vector with ∞ for variables in $\mathrm{Out}(\mathtt{C})$ and 0 for other variables.

Algorithm

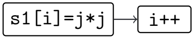
1. Pick a loop at top level.
2. Construct a **dependence graph**, which uses the DFG.
3. Compute its **condensation graph** from dependence graph.
4. Compute a **covering** of the condensation graph.
5. Create a loop per element of the covering.

W :=
```
while(t[i] > j) {
    s1[i]=j*j;  // C1
    s2[i]=1/j;
    i++;        // C3
}
```

$$\mathbb{M}(\mathtt{W}) = \begin{array}{c} \\ \mathtt{t} \\ \mathtt{i} \\ \mathtt{j} \\ \mathtt{s1} \\ \mathtt{s2} \end{array} \begin{array}{c} \begin{array}{ccccc} \mathtt{t} & \mathtt{i} & \mathtt{j} & \mathtt{s1} & \mathtt{s2} \end{array} \\ \begin{bmatrix} 1 & \infty & 0 & \infty & \infty \\ 0 & \infty & 0 & \infty & \infty \\ 0 & \infty & 1 & \infty & \infty \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\mathrm{In}(\mathtt{C}_1) = \{\mathtt{i}, \mathtt{j}\}$

$\mathrm{Out}(\mathtt{C}_3) = \{\mathtt{i}\}$



Definition: Dependence graph

The *dependence graph* of the loop $\mathtt{W} := \mathtt{while\ e\ do}\ \{\mathtt{C}_1; \cdots ; \mathtt{C}_n\}$ is the graph whose vertices is the set of commands $\{\mathtt{C}_1; \cdots ; \mathtt{C}_n\}$, and there exists a directed edge from $\mathtt{C}_i$ to $\mathtt{C}_j$ if and only if there exists variables $\mathtt{x} \in \mathrm{Out}(\mathtt{C}_j)$ and $\mathtt{y} \in \mathrm{In}(\mathtt{C}_i)$ such that $\mathbb{M}(\mathtt{W})(\mathtt{y}, \mathtt{x}) = \infty$.

Condensation Graph & Covering

Given a dependence graph, its *condensation graph* G_W is the graph whose

- vertices are strongly connected components (SCCs) and
- edges are the edges whose source and target belong to distinct SCCs.

We then find the *proper saturated covering* of G_W. For graph G,

- *covering* is a collection of subgraphs such that $G = \cup_{i=1}^{j} G_i$.
- *saturated covering* is a covering such that for all edges with source in G_i, its target belongs to G_i as well.
- It is *proper* if none of the subgraph is a subgraph of another.

Constructing Output

Lastly, we construct loop $\tilde{\texttt{W}}$ by inserting a loop for each element in the proper saturated covering.

If $\tilde{\texttt{W}}$ contains multiple loops, parallelize $\tilde{\texttt{W}}$.

Identify In and Out variables

```
while (j < m) {
  x = r[i] * A[i][j];    // C1
  y = A[i][j] * p[j];    // C2
  s[j] = s[j] + x;       // C3
  q[i] = q[i] + y;       // C4
  j++;                   // C5
}
```

$$\mathrm{Out}(\mathtt{C}_1) = \{\mathtt{x}\}$$
$$\mathrm{In}(\mathtt{C}_1) = \{\mathtt{A}, \mathtt{i}, \mathtt{j}, \mathtt{r}\}$$
$$\vdots$$
$$\mathrm{Out}(\mathtt{C}_3) = \{\mathtt{s}\}$$
$$\mathrm{In}(\mathtt{C}_3) = \{\mathtt{s}, \mathtt{j}, \mathtt{x}\}$$
$$\vdots$$
$$\mathrm{Out}(\mathtt{C}_5) = \{\mathtt{j}\}$$
$$\mathrm{In}(\mathtt{C}_5) = \{\mathtt{j}\}$$

Construct DFGs for each command

```
while (j < m) {
  x = r[i] * A[i][j];    // C1
  y = A[i][j] * p[j];    // C2
  s[j] = s[j] + x;       // C3
  q[i] = q[i] + y;       // C4
  j++;                   // C5
}
```

$$\mathbb{M}(\mathtt{C}_1) = \begin{array}{c} \\ \mathtt{i} \\ \mathtt{j} \\ \mathtt{m} \\ \mathtt{x} \\ \mathtt{y} \\ \mathtt{A} \\ \mathtt{r} \\ \mathtt{s} \\ \mathtt{p} \\ \mathtt{q} \end{array} \begin{array}{c} \begin{array}{cccccccccc} \mathtt{i} & \mathtt{j} & \mathtt{m} & \mathtt{x} & \mathtt{y} & \mathtt{A} & \mathtt{r} & \mathtt{s} & \mathtt{p} & \mathtt{q} \end{array} \\ \begin{bmatrix} 1 & \cdot & \cdot & \infty & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & \cdot & \infty & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \infty & \cdot & 1 & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \infty & \cdot & \cdot & 1 & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 1 \end{bmatrix} \end{array}$$

Compose DFGs of commands $\mathbb{M}(\texttt{C}_1;\dots;\texttt{C}_\texttt{n})$ and apply loop correction $E^t \times O$

$$
\mathbb{M}(\texttt{C}) = \begin{array}{c} \\ \texttt{i} \\ \texttt{j} \\ \texttt{m} \\ \texttt{x} \\ \texttt{y} \\ \texttt{A} \\ \texttt{r} \\ \texttt{s} \\ \texttt{p} \\ \texttt{q} \end{array}
\begin{array}{c}
\begin{array}{cccccccccc} \texttt{i} & \texttt{j} & \texttt{m} & \texttt{x} & \texttt{y} & \texttt{A} & \texttt{r} & \texttt{s} & \texttt{p} & \texttt{q} \end{array} \\
\begin{bmatrix}
1 & \cdot & \cdot & \infty & \infty & \cdot & \cdot & \cdot & \cdot & \infty \\
\cdot & \infty & \cdot & \infty & \infty & \cdot & \cdot & \infty & \cdot & \infty \\
\cdot & \infty & 1 & \infty & \infty & \cdot & \cdot & \infty & \cdot & \infty \\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \infty & \cdot & \cdot \\
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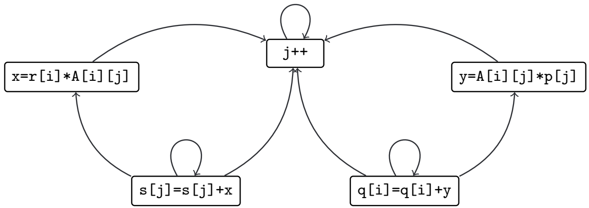
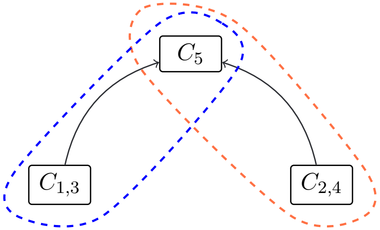
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \infty
\end{bmatrix}
\end{array}
$$

$\mathbb{M}(\texttt{C}) = \mathbb{M}(\texttt{C}_5) \times \cdots \times \mathbb{M}(\texttt{C}_1) + \mathrm{Corr}(\texttt{e})_\texttt{C}$

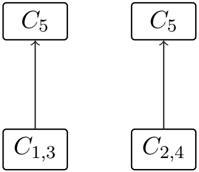
Construct a dependence graph. Vertices are the set of commands $\{\mathtt{C}_1; \cdots; \mathtt{C}_\mathtt{n}\}$. Add directed edge from $\mathtt{C}_i$ to $\mathtt{C}_j$ iff $\exists\, x, y$, where $\mathtt{x} \in \mathrm{Out}(\mathtt{C}_j)$ and $\mathtt{y} \in \mathrm{In}(\mathtt{C}_i)$ and $\mathbb{M}(\mathtt{W})(\mathtt{y}, \mathtt{x}) = \infty$.



Construct a condensation graph and proper saturated covering.



$\Rightarrow$



Example

Step 6 of 6

Distribute loops and parallelize.

$\tilde{\text{W}} :=$ parallel

```
while (j < m) {
  x = r[i] * A[i][j];
  s[j] = s[j] + x;
  j++;
}
```

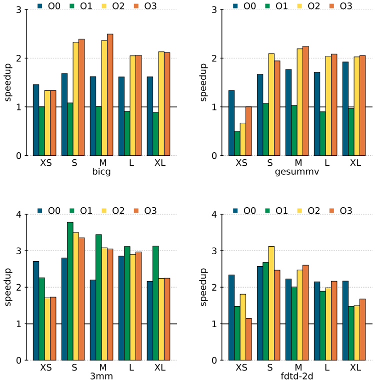
```
while (j < m) {
  y = A[i][j] * p[j];
  q[i] = q[i] + y;
  j++;
}
```

Experimental Evaluation

- Our artifact[1] is a collection of benchmarks.
- Mapped imperative syntax to `C` language.
- Used OpenMP directives to parallelize.
- Measured on standard benchmark suites, partially converted to `while` loops.
- Compared to an alternative loop transformation tool.

[1] Clément Aubert et al. *Distributing and Parallelizing Non-canonical Loops – Artifact*. Version 1.0. Sept. 2022. DOI: 10.5281/zenodo.7080145. URL: https://github.com/statycc/loop-fission.

Experimental Results



- Enables transformation and parallelization of loops ignored by alternative methods.
- Non-canonical loops: speedup upper-bounded by the number of parallelizable loops produced by transformation.
- Canonical loops: comparable to alternative methods in speedup potential.
- Demonstrated automatic insertion of parallel directives and practicality of this technique.

Conclusion

- Introduced an automatable loop optimization technique that adds parallelization potential to imperative programs.
- It is loop and language-agnostic – many possible applications.
- We presented the algorithm to perform the loop optimization.
- Experimental results demonstrate expected performance gain – see artifact
- See our paper for proof of preservation of semantic correctness.

statycc/loop-fission