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Classwork 1.1

1. Classify the following as either an ODE or PDE. Give the order, indicate the independent/dependent variables, and if it is linear or nonlinear

DE	ODE/PDE?	Order?	Indpt/Dep?	Linear/Nonlinear?
$\frac{5d^2}{dt^2} + 4\frac{dx}{dt} + 9x = 7\sin x$				
$y'' = (\sin x)\, y'$				
$u_{xx} + u_{yy} = 0$				
$y + y\left(\frac{d^3y}{dx^3}\right) = 17$				

2. Write a differential equation that fits the physical description
   a. The rate of change of the mass $z$of salt at time t is proportional to the square of the mass of salt present at time t.

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Classwork 1.2

1. Decide whether

$$y = \sin x + x^2$$

   is a solution to the DE

$$y'' + y = x^2 + 2$$

2. Determine whether

$$x^2 + y^2 = 4$$

   is a solution to the DE

$$\frac{dy}{dx} = \frac{x}{y}.$$

3. Determine whether Thm 1 applies that the given IVP has a unique solution.

$$\frac{dy}{dt} - ty = \sin^2 t\,, \qquad y(\pi) = 5.$$

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Classwork 1.3

1. A model for the velocity v at time t of a certain object falling under the influence of gravity in a viscous medium is given by the equation

$$\frac{dv}{dt} = 1 - \frac{v}{8}.$$

   a. Sketch the direction field for the DE
   b. From the direction field, sketch the solutions with initial conditions $v(0) = 5$.
   c. Identify the nullclines from the curve. Use this to answer the following question:
      i. Why is the value $v = 8$ called the "terminal velocity"?

2. a. Draw the isoclines with their direction markers and sketch several solution curves for

$$\frac{dy}{dx} = 2x^2 - y.$$

b. Sketch the curve satisfying $y(0) = 0$.

Classwork 1.4

1. a. Use Euler's method of approximation to solve the following IVP using step size $h = 0.1,\ N = 5$.

$$\frac{dy}{dx} = \frac{x}{y},\ \ y(0) = -1$$

   b. Show that the implicit equation

$$y^2 = x^2 + 1$$

   satisfies the IVP.

   c. Plot the polygonal-line numerical approximation in (a) together with the result in (b) using $N = 50$.

      i. What can you say about the error in the approximation scheme?

      ii. How can toy improve the approximation?

      iii. Use $h = 0.01$ and compare the graph of this case with the case where $h = 0.1$. Any improvement(s)?

2. a. Find the value of h for the Euler's method such that $y(1)$ is approximated to within ± 0.01, if $y(t)$ satisfies

$$y' = x - y$$

b. Also, find the value of x_0 to within ± 0.05, for the Euler's method such that $y(x_0) = 0.2$.

c. Graph the polygonal-line approximate solution, together with the exact solution

$$y(t) = e^{-x} + x - 1$$

for the choice of $h\ and\ x_0$ you picked.

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Classwork 2.2

1. Determine whether the given DE is separable.
   a. $\frac{dy}{dx} = 4y^2 - 3x + 1$
   b. $\frac{dx}{dt} = t\ln(s^{2t}) + 8t^2$

2. Solve the following equations.
   a. $x\frac{dy}{dx} = \frac{1}{y^3}$

b. $\frac{dy}{dx} = \frac{\sec^2 y}{1+x^2}$

c. $y^{-1}dy + ye^{\cos x}\sin x\ dx = 0, \qquad y\left(\frac{\pi}{2}\right) = 1.$

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Classwork 2.3

1. Which of the following DE is/is not a linear differential equation? Why?
   a. $y\frac{dy}{dx} + x^2 y = \sin x$

   b. $3t = e^t \frac{dy}{dt} + y \ln t$

2. Obtain the general solution to the following linear differential equation.
   a. $\frac{dy}{dx} - y - e^{3x} = 0$

b. $(t + y + 1)dt - dy = 0$

c. $\frac{dy}{dx} + \frac{3y}{x} + 2 = 3x, \quad y(1) = 1.$

d. $(\cos x)\frac{dy}{dx} + (\sin x)y = x^2 \cos x\,, \qquad y(0) = 1$

Classwork 2.4

1. Determine whether the following equations are exact. If so, solve them.
   a. $(\cos x \cos y + 2x)dx - (\sin x \sin y + 2y)dy = 0$

b. $\left(ye^{xy} - \frac{1}{y}\right)dx + \left(xe^{xy} + \frac{x}{y^2}\right)dy = 0$

c. $(\tan y - 2)dx + \left(x \sec^2 y + \frac{1}{y}\right) dy = 0, \qquad y(0) = 1.$

Classwork 2.5

1. Solve the following equations.

$$(2x)dx + (y^2 - 3x^2)dy = 0$$

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2. Find an integrating factor of the form $x^n y^m$ and solve the equation

$$(12 + 5xy)dx + (6xy^{-1} + 3x^2)dy = 0$$

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Classwork 2.6

1. Use the method discussed under "Homogeneous Equations" to solve.

$$(y^2 - xy)dx + (x^2)dy = 0$$

2. Use the method discussed under equation of form "$\frac{dy}{dx} = G(ax + by)$" to solve.

$$\frac{dy}{dx} = (x - y + 5)^2$$

3. Use the method discussed under "*Bernoulli equation*" to solve

$$\frac{dy}{dx} + y = e^x y^{-2}$$

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Classwork 3.2

1. A brine solution of salt flows at a constant rate of 6L/min into a large tank that initially held 50L of brine solution in which was dissolved 0.5kg of salt. The solution inside the tank is kept well stirred and flows out of the tank at the same rate. If the concentration of salt in the brine entering the tank is 0.05kg/L,
   a. determine the mass of salt in the tank after t mins.
   b. When will the concentration of salt in the tank reach 0.03kg/L?

2. If initially there are 50g of radio active substance and after 3 days there are only 10g remaining, what percentage of the original amount remains after 4 days?

Classwork 3.3

1. A warehouse is being built that will have neither heating nor cooling. To illustrate the effect insulation will have on the temperature inside the warehouse, assume the outside temperature varies as a sine wave, with a minimum of $16°C$ at 2:00AM and a maximum of $32°C$ at 2:00PM. Assuming the exponential term has died off,
   a. what is the lowest temperature inside the building if the time constant is 1hr? 5hr?
   b. What is the highest temperature inside the building if the time constant is 1hr? 5hr?

2. On a hot Saturday morning while people are working inside, the air conditioner keeps the temperature inside the building at $24°C$. At noon the air conditioner is turned off, and the people go home. The temperature outside is a constant $38°C$ for the rest of the afternoon. If the time constant for the building is 4hr, what will be the temperature inside the building
   a. at 2:00PM?
   b. At 6:00PM?
   c. When will the temperature inside the building reach $26°C$?

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Classwork 3.6

1. Show that when Trapezoidal method is used to approximate the solution of the initial vale problem

$$y' = y, \quad y(0) = 1$$

At $x = 1$, then we get

$$y_{n+1} = \left(\frac{1+\frac{h}{2}}{1-\frac{h}{2}}\right) y_n, \qquad n = 0,1,2,\cdots$$

which leads to the approximation $\left(\frac{1+\frac{h}{2}}{1-\frac{h}{2}}\right)^{\frac{1}{h}}$.

Compute the approximation for $h = 10^{-4}$ and compare your result with e.

2. Use the improved Euler's method subroutine with step $h = 0.1$ to approximate the solution to the initial value problem

$$y' = x - y^2, \qquad y(1) = 0$$

using 5 points.

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Classwork 4.2

1. (a) Verify that $y(t) = e^{-3t}\sin(\sqrt{3}t)$ is a solution to

$$y'' + 6y' + 12y = 0.$$

(b). What is $\lim_{t\to\infty} y(t)$?

2. Find a synchronous response of the mass-spring oscillator of the form $A\cos\omega t + B\sin\omega t$ satisfying

$$y'' + 2y' + 4y = 6\cos 2t + 8\sin 2t.$$

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Classwork 4.2

1. a). Find a general solution to the differential equation

$$y'' + 4y' + 2y = 0.$$

b). Find a UNIQUE solution to the differential equation

$$y'' + 4y' + 2y = 0, \quad y(0) = 0, \; y'(0) = 1.$$

c). What is $\lim_{t \to \infty} y(t)$?

2. a). Find two independent solutions to the differential equation

$$y'' - 6y' + 9y = 0$$

b. Find the characteristic equation and the eigenvalue associated with the DE.

c). Find a general solution to the DE.

b). Find a UNIQUE solution to the differential equation

$$y'' - 6y' + 9y = 0, \quad y(0) = 0, \;\; y'(0) = 1.$$

3. Determine whether the functions $y_1(t) = e^{2t}\ and\ y_2(t) = te^{2t}$ are linearly independent or dependent on any interval (a, b).

4. Determine whether the functions $y_1(t) = e^{2t}, y_2(t) = e^{3t},\ and\ y_3(t) = e^{4t}$ are linearly independent or dependent on any interval (a, b) by first finding their Wronskian.

5. Find a general solution to the 3rd order differential equation

$$y''' + y'' - 6y' + 4y = 0.$$

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Classwork 4.3

1. a). Solve the given initial value problem

$$y'' - 4y' + 2y = 0, \quad y(0) = 0, \;\; y'(0) = 1.$$

   b). What does the solution curve represent?

   c). Sketch the solution.

   d). What is $\lim_{t\to\infty} y(t)$?

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Classwork 4.4

a). Find the homogeneous solution $y_h(t)$
b). Find a particular solution $y_p(t)$
b). the general solution $y(t)$
c). the exact solution to the following differential equations.

1. $y'' - 3y' - 4y = 2t^2, \; y(0) = 1, \; y'(0) = 2.$

2. $y'' - 2y' + y = 3te^t, y(0) = 1,\ y'(0) = 2$

3. $y'' + 2y' + 4y = \sin(2t)\,, y(0) = 1,\ y'(0) = 2$

4. $y'' + 2y' + 4y = e^{-t}\sin(\sqrt{3}t)\,, y(0) = 1,\ y'(0) = 2$

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Classwork 4.5

a). Find the homogeneous solution $y_h(t)$
b). Find a particular solution $y_p(t)$
c). the general solution $y(t)$
d). the exact solution to the following differential equations.

1. $y'' + 2y' + 4y = 3t + 2\sin 2t\,,\;\; y(0) = 1,\; y'(0) = 2$

2. $y'' - 2y' - 3y = 3t^2 - 5,\ y(0) = 1,\ y'(0) = 2.$

3. $y'' - y' - 2y = \cos t - \sin 2t$, $y(0) = -7/20$, $y'(0) = 1/5$

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Classwork 4.6

a). Find the homogeneous solution $y_h(t)$

b). Find a particular solution $y_p(t)$

c). the general solution $y(t)$

d). the exact solution to the following differential equations.

1. $y'' + y = \sec t\,,\ \ y(0) = 1,\ y'(0) = 2$

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2. $y'' + 5y' + 6y = 18t^2, \; y(0) = 1, \; y'(0) = 2$

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Classwork 4.7

a). Find the homogeneous solution $y_h(t)$
b). Find a particular solution $y_p(t)$
c). the general solution $y(t)$
d). the exact solution to the following differential equations.

1. $t^2y'' + ty' - y = 0,\ \ y(1) = 1,\ \ y(2) = 3$

2. $t^2y'' + ty' + 9y = -\tan(3\ln t)$

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Classwork 7.2

1. Use Definition 1 to determine the Laplace transform of the following functions.
   a. $f(t) = t$

   b. $f(t) = t^2$

   c. $f(t) = t^n$

d. $f(t) = t^2 e^{5t}$

e. $f(t) = e^{-t} \sin 3t$

f. $f(t) = \begin{cases} 1 & if \ 0 < t < 2 \\ t & if \ 2 < t \end{cases}$

2. Use the Laplace Transform table and the linearity of Laplace Transform to find $\mathcal{L}[f(t)]$

$$\mathcal{L}[6e^{-2t} - t^2 + e^{5t}\cos 7t + t^2 e^{3t} - 8]$$

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Classwork 7.3

1. Determine the Laplace transform of the following functions.
   a. $f(t) = t^2 + e^t \sin 5t$

   b. $f(t) = t^2 e^{-t} - \sin^2 t$

c. $f(t) = \sin 3t \cos 3t$

d. $f(t) = te^{4t} \cos 5t$ Hint: Use Theorem 6.

2. Use Theorem 1 and Theorem 5 to find the Laplace $\mathcal{L}\{y\}$ for the differential equation

$$y'' + 3y' + 6y = t^3, \qquad y(0) = 3, y'(0) = 1.$$

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Classwork 7.4

1. Evaluate

a. $\mathcal{L}^{-1}\left\{\frac{2s+16}{s^2+4s+8}\right\}$

b. $\mathcal{L}^{-1}\left\{\frac{6}{(3s-4)^4}\right\}$

c. Evaluate $\mathcal{L}^{-1}\left\{\frac{4s+1}{(s+1)(s-2)(s-1)}\right\}$

Solution: This is an example of case 1a. Why?

d. Evaluate $\mathcal{L}^{-1}\left\{\frac{4s+1}{(s-1)^2(s-2)}\right\}$

Solution: This is an example of case 1b. Why?

e. Evaluate $\mathcal{L}^{-1}\left\{\frac{2s^2+s}{(s^2-2s+5)(s-1)}\right\}$

Solution: This is an example of case 1c. Why?

2. $Part$ 1: Use Theorem 1 and Theorem 5 to find the Laplace $\mathcal{L}\{y\}$ for the differential equation

$$y' + 6y = 5, \qquad y(0) = 3.$$

Part II: From your result, use the inverse Laplace transform to solve for $y(t)$

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Classwork 7.5

1. Solve the IVP:

a. $y'' - 4y' + 5y = 4e^{3t}, \quad y(0) = 2,\ y'(0) = 7$

b. $y'' + 9y = 4t, \quad y(0) = 0, \; y'(0) = 7$

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c. $y'' + 3ty' - 6y = 1, \quad y(0) = 0,\ y'(0) = 0$

d. $y''' + 3y'' + 3y' + y = 0, \quad y(0) = -4,\ y'(0) = 4, y''(0) = -2.$