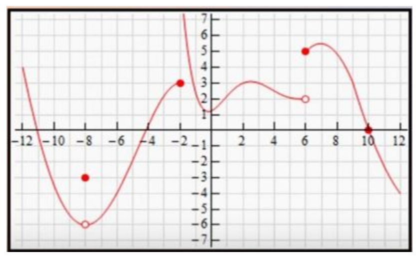
# MATH 2011- Calculus & Analytic Geom I
# Exam 1: Fall 2021 Practice Questions

Answer all questions. Name.....................................

You need to show your work in great details in order to earn full point.

Just writing the final answer where you are supposed to show full work will earn you little to no points atall.

**1.** (**20pts**) Evaluate the following limits using the graph of the function provided.



(a). $\lim_{x\to -8^-} f(x)$ (b). $\lim_{x\to -8^+} f(x)$ (c). $\lim_{x\to -8} f(x)$ (d). $\lim_{x\to -2^-} f(x)$ (e). $\lim_{x\to -2^+} f(x)$

(f). $\lim_{x\to -2} f(x)$ (g). $\lim_{x\to 6^-} f(x)$ (h). $\lim_{x\to 6^+} f(x)$ (i). $\lim_{x\to 6} f(x)$ (j). $\lim_{x\to 10^-} f(x)$

(k). $\lim_{x\to 10^+} f(x)$ (l). $\lim_{x\to 10} f(x)$ m). $f(-8)$ n). $f(-2)$ o). $f(6)$ p). $f(10)$

(q). Using the **definition of continuity only**, state if the function is left, right, removable, simply continuous or discontinuous at the points $x = -8, -2, 6$ and 10. **Explain why.**

(r). Do we have any removable discontinuities? If so, list them.

**2.** (**10pts**) If a ball is thrown into the air with a velocity of 10 ft/s, its height in feet $t$ seconds later is given by $s(t) = 10t - 1.5t^2$.

a). Estimate the instantaneous velocity at $t = 2$. (**Show appropriate table for this**)

b). Using your result in part (a), what is the slope of the tangent line to the curve at $t = 2$?

c). Find the equation of the tangent line at time $t = 2$.

d). What is the difference between a tangent line and a secant line? Use examples if appropriate.

**3.** (**15pts**) Find the exact value of the following limits. **Show your work to earn full point**.

a) $\lim\limits_{x\to 3} \frac{x^2-x-6}{x^2-5x+6}$

b) $\lim\limits_{x\to 0} \frac{\sqrt{x+4}-2}{x}$

c) $\lim\limits_{x\to 5} \frac{\frac{1}{x}-\frac{1}{5}}{x-5}$

d). When is a function $f(x)$ said to be continuous at a point $x = a$? Give the limit definition.

e). Is the function $f(x) = \frac{x^2-x-6}{x^2-5x+6}$ in part (a) continuous at the point $x = 3$? Justify using the **definition of continuity**. If not, answer part (e) below.

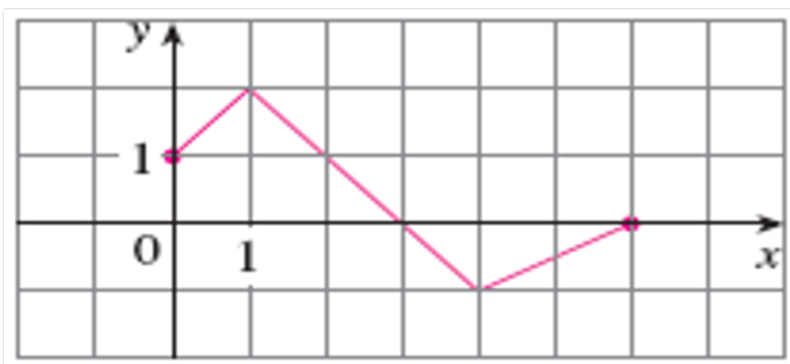
f). Explain how to make the function continuous if discontinuity is removable at $x = 3$.

**4.** (**10pts**) Given that $\lim\limits_{x\to -2} f(x) = 3$ and $\lim\limits_{x\to -2} g(x) = -4$, evaluate, using limit laws,

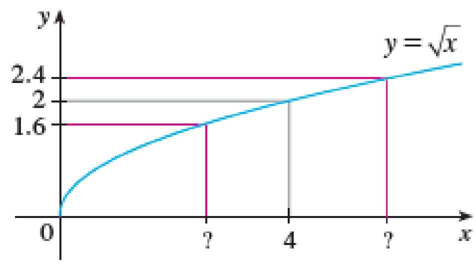
i). $\lim\limits_{x\to -2} \left(\sqrt{f(x)+1}\right)$ ii). $\lim\limits_{x\to -2} \frac{g(x)}{x^2}$ iii). $\lim\limits_{x\to -2} \frac{xf(x)+2f(x)}{(x+2)g(x)}$

**5.** (**10pts**) Use the graph of $f(x)$ below to sketch the following graph. Label graphs in (a), (b), (c), (d) as g(x), h(x), p(x), q(x), respectively. **Clearly calibrate your graph and show points corresponding to each vertices and endpoints of your graph**.

a). $y = f(x) - 1$ b). $y = f(x-2)$ c). $y = f(2x)$ d). $y = f^{-1}(x)$, inverse of $f(x)$



**6.** (**10pts**) Use the graph of $f(x) = \sqrt{x}$ below to find a number $\delta$ such that if $0 < |x - 4| < \delta$, then $|\sqrt{x} - 2| < 0.4$. **Show how you arrive at your final answer. Leave your answer in radical form.**




**7.** (**10pts**) Use the Intermediate Value Theorem (IVT) to show that a root of the equation $x^3 + 1 = 3x$ is in the interval $[1, 2]$. **Justify your answer and state all conditions used.**

**8.** (**15pts**) Evaluate the limits (**show your work in great details**)

a). $\lim_{x\to\infty} \frac{4x-5x^2-3x^3}{6x^3+1}$

b). Estimate the value of $\lim_{x\to\infty} \left(1 - \frac{1}{x}\right)^x$ to six decimal places.

c). $\lim_{x\to\infty} \sqrt{x^2 - 5x} - x$

d). $\lim_{x\to-\infty} \frac{5x^3+4x}{\sqrt{16x^6+2x}}$

**MATH 2011-CALCULUS & ANALYTIC GEOMETRY I**
**FALL 2021**
**EXAM II**

**<u>Answer all questions.</u>** Name_______________

1. **(15pts)** a. Write the formula for the limit definition of the derivative $f'(x)$
   b. Use **the limit-definition** to find the derivative $f'(x)$ of the function $f(x) = x^2 - 3x + 2.$

   c. Find the equation of the tangent line to the curve at point $a = 3.$

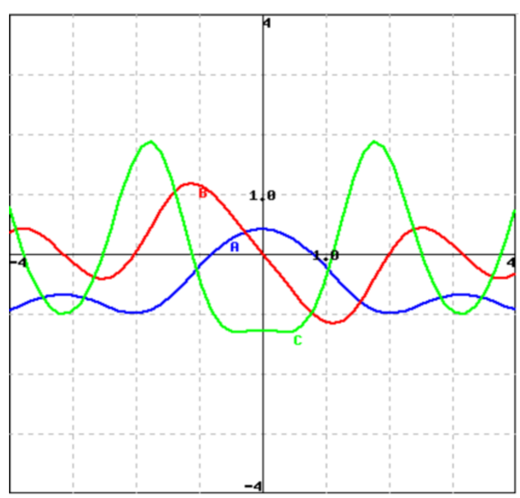
2. **(10pts)** Given the curve satisfying the equation $2xy + 4x + 5y^2 = 5.$
   a. Find $\frac{dy}{dx}$ using implicit differentiation.
   b. Find the point on the curve where the tangent line to the curve is horizontal.

3. **(25pts)** Differentiate the following functions using appropriate differentiation rule. **Simplify appropriately in order to earn full point.**

   a. $f(x) = \frac{3x^2+2\sqrt[4]{x}}{x^3}$
   b. $f(x) = (5x^3 + 2x + \tan x)(5x^{\frac{7}{5}} - \sqrt[3]{x^2})$
   c. $f(x) = (5x^2 + x + 3)^{20}$
   d. $f(x) = \frac{x^2+5x}{3x^2+1}$
   e. $f(x) = 7^{(2x^2+1)}$

4. **(10pts)** Use appropriate method to find the derivative of $f(x) = \frac{(3x^2-1)^9(2x^3+5x-2)^5}{(e^x+5)^{11}}$

5. **(10pts)** The three graphs to the right represent $f(x), f'(x)\ and\ f''(x)$. Identify them accordingly **with appropriate justification.**



6. **(10pts)** A particle moves according to a law of motion, $s(t) = t^3 - 8t^2 + 24t, t \geq 0$, where $t$ is measured in seconds and $s$ in feet
   a. What is the velocity after 2 secs?
   b. When is the particle at rest?
   c. Find the acceleration after 1 sec.
   d. When is the particle speeding up?

7. **(10pts)** Suppose $y = \sqrt{4x - 4}$, where *x* and *y* are functions of $t$. If $\frac{dx}{dt} = 3$, find $\frac{dy}{dt}$ when $x = 5$.

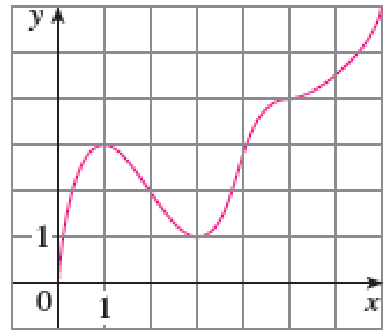
8. **(10pts)** a). Find the linearization of the function $f(x) = \sqrt{x}$ at the point a=16.
   (b). Use this to find an approximation to $\sqrt{15.95}$ to four decimal places.

MATH 2011: **EXAM 3** Name:_______________

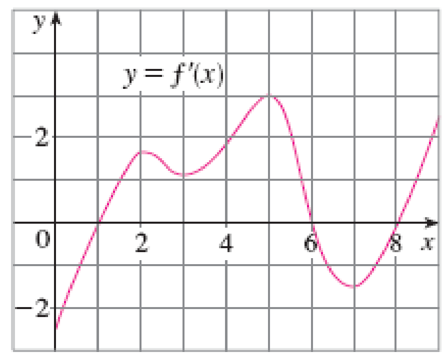
**FALL 2021**

Answer all questions. Show all work in order to earn full point.

1. **(15pts)** Using the graph of $f(x)$ to the right, find
   a) Critical point(s) of $f(x)$
   b) Interval(s) where $f(x)$ is increasing and decreasing
   c) Local maximum and local minimum value of $f(x)$
   d) Absolute maximum and local minimum value of $f(x)$
   e) Point of inflection of $f(x)$
   f) Interval(s) where $f(x)$ is concave up and concave down




2. **(15pts)** Using the graph of $f'(x)$ to the right, find
   a) Critical point(s) of $f(x)$
   b) Interval(s) where $f(x)$ is increasing and decreasing
   c) Local maximum and local minimum value of $f(x)$
   d) Point of inflection of $f(x)$
   e) Interval(s) where $f(x)$ is concave up and concave down




3. **(25pts)** Given the function $f(x) = x^3 - 3x^2 - 9x + 4$. Find
   a) Critical point(s) of $f(x)$
   b) Interval(s) where $f(x)$ is increasing and decreasing
   c) Local maximum and local minimum value of $f(x)$
   d) Absolute maximum and absolute minimum value of $f(x)$ on the interval [-2, 2].
   e) Point of inflection of $f(x)$
   f) Interval(s) where $f(x)$ is concave up and concave down

4. **(15pts)** Use L'Hopital rule (if applicable, else write N/A and use other method) to find the limit

a) $\lim_{x \to 0} \frac{2x}{1-\cos x}$

b) $\lim_{x \to 0} \frac{4^x - 2^x}{x}$

c) $\lim_{x \to \infty} \left(1 - \frac{3}{x}\right)^x$

5. **(15pts)** The equation

$$e^x = 4 - x^2$$

has two solutions in the interval $[-2.5, -1.5]$ and $[0.5, 1.5]$.

a). Use Newton's method to find an approximate solution in the interval $[0.5, 1.5]$ to 8 decimal places.

b). Use Newton's method to find an approximate solution in the interval $[-2.5, -1.5]$ to 8 decimal places.

6. **(15pts)** Find the dimensions of a rectangle with perimeter $120m$ whose area is as large as possible by answering the following questions

   a. State the objective function.
   b. State the constraint(s).
   c. Use your constraint to reduce objective function to function of one variable only.
   d. State the interval of optimization.
   e. Solve the optimization problem.

**MATH 2011** **Final Exam**

**Fall 2021**

**Answer all questions.** Name:__________________

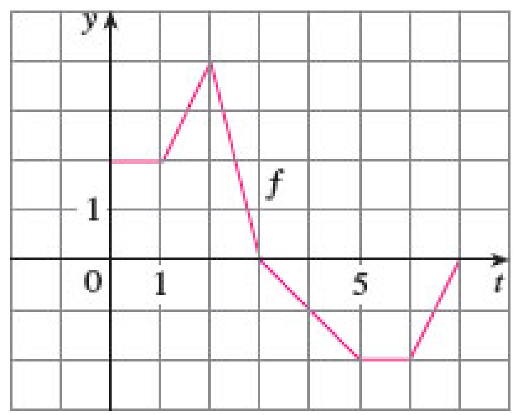
1. **(15pts)** Evaluate the general indefinite integral
   a. $\int 1 + \sqrt[4]{x^5} + \frac{3}{7}x^2\, dx$
   b. $\int \frac{t^3+5t^2+t}{3t^2}\, dt$
   c. $\int 2^x + \sec^2 x\ dx$

2. **(20pts)** Evaluate the definite integral using substitution method
   a. $\int_2^3 x^2\sqrt{x^3+1}\ dx$
   b. $\int_0^1 (3x^2+6x+1)^6(x+1)\ \ dx$
   c. Use result in (b) to find the area under the curve $y = (3x^2+6x+1)^6(x+1)$ over $[0,1]$.

3. **(20pts)** Evaluate
   a. $\int_0^3 f(t)\ dt$
   b. $\int_2^7 f(t)\ dt$

   Let $g(x) = \int_0^x f(t)\ dt$, where f is the whose graph is shown.

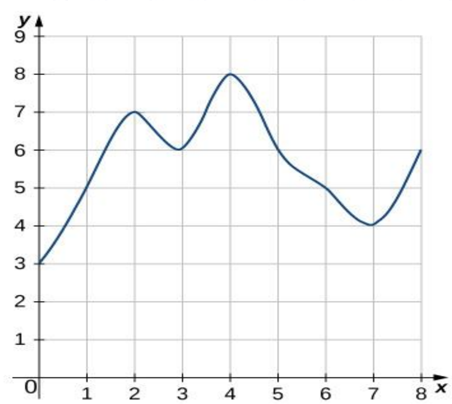
   c. On what interval is $g(x)$ increasing?
   d. Where does $g(x)$ have a maximum value?




4. **(20pts)** a). Draw rectangles for right endpoint approximation $R_4$ and **estimate $R_4$.**

   b). Draw rectangles for left endpoint approximation $L_4$ and **estimate $L_4$.**

   c). Draw rectangles for midpoint endpoint approximation $M_4$ and **estimate $M_4$**




5. **(15pts)** Given that $f(x) = x^2 + 2x$ on $[0, 3]$.
   a. Evaluate the area under the curve by computing $\lim_{n\to\infty} R_n$
   **b.** Compute $\int_0^3 x^2 + 2x$ dx and compare result with part (a). **Discuss your result.**

6. **(10pts)** The equation

$$x^3 = 4 - x^2$$

has a solution in the interval $[1,2]$.

Use Newton's method to find

a). the first four approximations $x_1, x_2, x_3, and\ x_4$

b). an approximate solution in the interval $[1,2]$ that is **<u>EXACT</u>** to 8 decimal places.

7. **(15pts)** Given the function $f(x) = 2x^3 - 6x^2 - 18x + 4$
   a. Find the critical point of f
   b. Find region where $f(x)$ is increasing or decreasing.
   c. Use this to find the local minimum and minimum values of $f(x)$
   d. Find the point of inflection of $f(x)$.
   e. Find region where f(x) is concave up and concave down
   f. Use information in parts (a)-(e) to sketch the graph of $f(x)$.

8. **(20pts)** Find the derivative of the following functions.

   a. $f(x) = \frac{x^2+5x}{e^x}$

   b). $f(x) = (x^2 - 1)e^x$

   c). $f(x) = (x^2 + \cos x)^{10}$

   d). $f(x) = \frac{(x+1)^7(3x^2+1)^9}{(\sin x+4x)^{11}}$

9. **(15pts)** Find the dimensions of a rectangle with **area** $16m^2$ whose **perimeter** is as small as possible by answering the following questions

   I. State the objective function.
   II. State the constraint(s).
   III. Use your constraint to reduce objective function to function of one variable only.
   IV. State the interval of optimization.
   V. Solve the optimization problem.

$$\sum_{i=1}^{n} c = c * n$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2} = \frac{n^2}{2} + \frac{n}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} = \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$$

$$\sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4} = \frac{n^4}{4} + \frac{n^3}{2} + \frac{n^2}{4}$$

$$\Delta x = \frac{b-a}{n}$$

$$R_n = \sum_{i=1}^{n} f(a + i * \Delta x)\, \Delta x$$

$$L_n = \sum_{i=0}^{n-1} f(a + i * \Delta x)\, \Delta x$$

$$M_n = \sum_{i=1}^{n} f\left(a + \left(i - \frac{1}{2}\right) * \Delta x\right) \Delta x$$